MEASUREMENT OF LONGITUDINAL IMPEDANCE AT KEKB

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Abstract

The current dependence of the synchronous phase and the bunch length was measured to estimate the longitudinal coupling impedance of KEKB. The bunch length was reduced to 4 mm owing to a negative momentum compaction lattice. The loss factor was measured at such a short bunch length. The bunch lengthening was observed even with a negative compaction in a high-current region, which suggests a possible microwave instability in the low energy ring.

1. INTRODUCTION

KEKB [1,2] is a multi-bunch, high-current, electron/positron collider for B meson physics. The collider consists of two storage rings: the Low Energy Ring (LER) for a 3.5-GeV positron beam and the High Energy Ring (HER) for 8-GeV electrons. Both rings store more than 1,000 bunches, where the harmonic number is 5120 with an RF frequency of 509 MHz.

Bunches are squeezed at an interaction point (IP) to gain the luminosity. The bunch length should be shorter than the beta function at the IP to avoid the hourglass effect. However, the bunch length increases with the bunch current due to inductive impedance under the lattice with positive momentum compaction. On the other hand, when the lattice is switched to negative momentum compaction, bunch shortening is expected due to the negative impedance. When the bunch length is short, however, it is expected to increase the loss factor. Reducing the loss factor is an important issue for KEKB.

The impedance had already been measured in the early stage of commissioning [3]. Some RF cavities were added and some vacuum components were modified after the previous measurement. Measuring the impedance is necessary not only to understand the current machine, but also to design a future machine, such as SuperKEKB [4], where a bunch length of 3 mm is expected.

2. LONGITUDINAL BEAM BEHAVIOR IN POSITIVE AND NEGATIVE MOMENTUM COMPACTION

KEKB adopts a unique lattice composed of the noninterleaved 2.5π cell structure [1]. The lattice has a wide dynamic aperture and flexibility for adjusting the parameters of the emittance and the momentum compaction. The momentum compaction, defined by \( \alpha = (\Delta C/C)/(\Delta p/p) \), where \( C \) is the circumference and \( p \) is the momentum, is determined by integrating the dispersion function at the bending magnets. The current lattice uses a positive momentum compaction factor of \( \alpha = 3.4 \times 10^{-4} \) for both LER and HER. The lattice was smoothly switched to use a negative compaction factor without changing the absolute value in both rings. Thus, the natural bunch length and the synchrotron tune would not be changed by switching.

Let us consider the longitudinal behaviour of a bunch. Accelerating cavities in a storage ring give energy to a circulating bunch to compensate for the radiation loss and parasitic loss. The synchronous phase of a bunch is determined by the balance between the cavity voltage and the total energy loss. The parasitic loss is caused by the resistive impedance of the environment, represented by the loss factor \( k(\alpha) \), depending on the bunch length. The shift in the synchronous phase due to the parasitic loss \( \Delta \phi_s \) is given by

\[
V_c \sin(\phi_{s0} + \Delta \phi_s) = \frac{U_0}{e} + k(\alpha) \frac{T_0}{I_b},
\]

where \( V_c \) is the cavity voltage, \( \phi_{s0} \) is the synchronous phase of a zero-current limit, \( U_0 \) is the radiation loss per turn, and \( T_0 \) is the revolution period. When the phase shift \( \Delta \phi_s \) is small enough compared with unity, it is given by

\[
\frac{\Delta \phi_s}{I_b} = k(\alpha) \frac{T_0}{V_c \cos \phi_{s0}}.
\]

As illustrated in Fig. 1, a bunch rides on the descending shoulder of an RF wave in the case of \( \alpha > 0 \). On the other hand, the synchronous phase moves to the rising shoulder in the case of \( \alpha < 0 \), to maintain the phase stability. The synchronous phase shifts to compensate the additional energy loss due to the parasitic loss. Thus, the beam phase advances in the case of \( \alpha > 0 \) and lags in the \( \alpha < 0 \) on the RF wave, as the bunch current increases. The rates of the phase shift are proportional to the loss factor.

Figure 1: Schematic of the synchronous phase of a bunch in positive and negative momentum compaction.

The natural bunch length is determined by the lattice parameters at a zero current. However, the bunch shape is determined by its own potential-well field at finite bunch current. Inductive or capacitive impedance of the environment changes the bunch length due to the potential-well distortion. When a purely inductive impedance is assumed in the case of \( \alpha > 0 \), bunch lengthening is analytically derived [3], and given by

\[
\left( \frac{\sigma}{\sigma_{10}} \right)^3 - \left( \frac{\sigma}{\sigma_{10}} \right) = \frac{e d}{\pi} \frac{R}{4 \sqrt{\pi \gamma_0} E} \frac{Z_1(\omega)}{n}.
\]
Here, $R$ is the average radius of a ring, $Z(t, \omega)$ is the inductive impedance and $n$ is an integer given by $n = a/\omega_0$, where $\omega_0$ is the angular revolution frequency. In the case of $\alpha < 0$, the negative inductance causes bunch shortening. However, when the bunch length is too short, bunch shortening might occur, because an effect of negative capacitance appears due to the extended spectrum of a short bunch. Since two effects of lengthening and shortening are actually mixed, variations of the bunch length might be small. When the bunch current increases further, a microwave or turbulent instability takes place above the threshold in both cases of $\alpha > 0$ and $\alpha < 0$, which results in increasing the energy spread as well as the bunch lengthening.

3. MEASUREMENT

3.1 Beam Phase

The beam phase was measured turn by turn with a four-dimensional beam-position monitor [5]. Two amplitudes of an $I/Q$ (in-phase and quadrature phase) demodulator working at an RF frequency give a phase difference between the beam and the RF phase. Assuming that the RF phase is constant, we can obtain a relative beam phase. We confirmed that an average phase was settled within ±0.1° under a stable beam.

The measurement was performed with a single bunch. Figure 2 shows variations of the measured beam phase as a function of the bunch current in the LER. The beam phase is advancing, as the bunch current increases, in the case of $\alpha > 0$, and it is lagging in the case of $\alpha < 0$. Assuming that the RF phase is constant, we can estimate the synchronous phase, $\varphi_{s0}$ from an extrapolated phase to zero current. The estimated $\varphi_{s0}$ is 10.3° at the LER and 16.7° at the HER, respectively. On the other hand, the calculated $\varphi_{s0}$ is 11.8° at the LER and 15.5° at the HER, the calculation is consistent with the estimation. We find that the slope in $\alpha < 0$ is steeper than that in $\alpha > 0$ at both rings. The result suggests that the loss factor in the case of $\alpha < 0$ is larger than that of $\alpha > 0$.

Figure 2: Beam-phase vs. bunch current in the LER, (a) $\alpha > 0$ and (b) $\alpha < 0$.

3.2 Bunch Length

The bunch length was measured using a method based on the beam spectrum. A bunch-length monitor detecting two frequency components in the beam spectrum under the condition of $\omega_0 \sigma_r < 1$, where $\omega_0$ is the detected angular frequency and $\sigma_r$ is the rms bunch length in time, indicates the rms length of a bunch in real-time, even if the bunch shape changes [6]. The monitor detects two frequency components of 1.02 GHz and 2.54 GHz and has a resolution of ±0.2 mm at a bunch length of 4 mm. Since the gains between two channels are not always the same, we need to calibrate the detector in order to obtain an absolute value of the bunch length. The calibration is performed based on the assumption that an extrapolated bunch length to zero current approaches the natural bunch length. The monitor detects an average value of the bunch length, since the monitor does not have a function to select a specific bunch.

The measurement was performed with a multi-bunched beam, where bunches with an equal intensity are separated by 320 ns. Since bunches are well separated, the effect of a coupled bunch would be negligible. Figure 3-(a) and (b) show the average bunch length as a function of the bunch current with the two cases of $\alpha > 0$ and $\alpha < 0$. The bunch length increases with the bunch current in $\alpha > 0$ and the bunch length in $\alpha < 0$ is shorter than that in $\alpha > 0$ at both rings. A bunch length of around 4 mm is obtained at a low-current region at the LER; however, it increases as well as the case of $\alpha > 0$, when the current is above 0.4 mA. At the HER, the bunch length keeps around 5 mm in $\alpha < 0$, even at a high-current region. Thus, a difference in the bunch length between $\alpha > 0$ and $\alpha < 0$ expands, as the bunch current increases.

![Figure 3](image-url)

Figure 3: Bunch length as a function of the average bunch current, (a) dots in the cases of $\alpha > 0$ and squares $\alpha < 0$ at the LER, (b) dots in the cases of $\alpha > 0$ and squares $\alpha < 0$ at the HER. The arrows indicate the natural bunch length, 4.74 mm at the LER and 5.22 mm at the HER.
4. DISCUSSION

4.1 Possibility of the Microwave Instability

As shown in Fig. 3-(a), unexpected bunch lengthening was observed in the case of $\alpha < 0$ at the LER, compared with that in the HER. A microwave instability is suspected in the LER. First, The Boussard criterion is considered. The threshold current is estimated to be 0.26 mA at the LER, using an impedance of $Z_i/n = 0.07 \Omega$ [3]. However, the equation might be underestimated [7], since it is based on the coating beam theory. Secondly, A. Chao and J. Gareyte have derived a scaling law to explain the microwave instability [8]. The bunch length satisfies the scaling property, $\sigma_z = F(\xi)$, with

$$\xi = \frac{\alpha I_b}{\nu^2 E}.$$

We find that the parameter $\xi$ does not depend on $\alpha$. On the other hand, an experimental result revealed that the measured bunch lengths for two values of $\alpha$ were nearly equal at a high bunch current. Thirdly, the measured bunch length in the case of $\alpha > 0$ was fitted by the calculated bunch length with purely inductive impedance, as shown in Fig. 4. We find that the bunch length deviates from the calculated bunch length above a bunch current of 0.6 mA. However, we do not yet have measured increase of the energy spread. No coherent oscillation was observed even above a bunch current of 1.0 mA.

Figure 4: Measured bunch length indicated by dots in the case of $\alpha > 0$. The line is the calculated bunch length as a function of the bunch current at the LER, assuming $Z_i/n = 0.06 \Omega$.

4.2 Impedance

The loss factor obtained from the slope in the beam phase is plotted in Fig. 5, together with the data measured in 1999. The inductive impedance estimated from the bunch lengthening is represented in Table 1. Comparing the data obtained in 2003 with those in 1999, the loss factor does not indicate a large change. It is noticed that a loss factor increases and tends to reach 100 V/pC at a bunch length of 4 mm. Though the inductive impedance tends to slightly reduce, the inductance at the LER is still 4-times larger than the design value [1]. During 4 years, many components of both rings have been modified. Among the modifications, the number of normal conducting cavities has increased to store a high current beam, which may contribute to increasing the loss factor. We find that the loss factor slightly increases in the HER. The movable collimators have been improved [9], which has reduced the impedance. The effects of each modification, however, are not reflected in the measurement.

Figure 5: Loss factor as a function of the bunch length, (a) LER and (b) HER. The crosses are the values measured in 1999 and the dots are in 2003.

Table 1: Inductive impedance.

<table>
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<tr>
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<th>1999</th>
<th>2003</th>
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<tbody>
<tr>
<td>LER $Z_i/n$ ((\Omega))</td>
<td>0.072 ± 0.011</td>
<td>0.060 ± 0.01</td>
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<tr>
<td>HER $Z_i/n$ ((\Omega))</td>
<td>0.076 ± 0.006</td>
<td>0.065 ± 0.006</td>
</tr>
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REFERENCES