Robinson Instability under Cavity Voltage Feed-back Observation at the SPring-8 Storage Ring

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1 Introduction

The observation of the Robinson instability under cavity voltage feed-back loop was performed at the SPring-8 storage ring and the results were compared with the analysis in [1]. First, we describe the application of the theory of the Robinson instability under cavity feed-back loops based on the reference [?] to The SPring-8 storage ring and show the results of the calculation. Then we show the results of the observation and the comparison with the calculated results.

2 Calculation for the SPring-8 Storage Ring

The parameters of the SPring-8 storage ring and symbols used in this paper are shown in Table 1. The SPring-8 storage ring has three RF stations(B,C,D). Each station has one klystron, eight single-cell RF cavities and two feed-back loops, one for amplitude and one for phase of sum of voltages of cavities.

Table 1
SPring-8 Parameters

Electron Energy	E_0	7.975	GeV
RF Frequency	f_{RF}	508.58	MHz
Harmonic Number	h	2436	
Radiation Loss/Turn	U_0	8.91	MV
Acceleration Voltage/Turn	V_c	12	MV
Max. Stored Current	I_0	100	mA
Synchrotron Frequency	f_s	~ 1.5	kHz
Number of RF stations	N_s	3	
Number of Cavities/Station		8	
Cavity Parameters			
Shunt Impedance	R_c	3	MΩ
Unloaded Q	Q_c	40000	
Coupling Factor	β	2	
Detuning Phase Angle	$\Delta\psi$	-5	deg.

2.1 Static Voltages in Cavity

We have to know the impedance of the cavity by solving the phaser diagram of the static voltages in a cavity. Every cavity in each station has the same parameters and the sum of these cavity voltage is controlled by two feed-back loops, one for amplitude and one for phase. Thus, we treat them as one cavity whose impedance is 8 times of a actual single cavity.

A cavity has a resonator type impedance of the form

$$Z(\omega) = R_L \cos\psi e^{i\psi}, \quad \tan\psi = Q_L\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right) \quad (1)$$

where ω_r is the resonant frequency and $Q_L = Q_c/(1+\beta)$, $R_L = R_c/(1+\beta)$. ω_r is dynamically controlled by a tuner control system and β is a coupling factor shown in Table 1.

The static part of phasers of cavity voltages are shown in Fig. 1 where $\tilde{i}_b = -2I_0$ and $\tilde{V}_b = Z(\omega_{RF})\tilde{i}_b = -2I_0 R_L \cos\psi e^{i\psi}$ are the static part of the beam current[2] and the beam induced voltage,respectively, and I_0 is the stored current. We do not show the suffix 0 which was used to tag static part of parameters in [1] and we use ϕ instead of ϕ_c in [1].

The resonant frequency ω_r is controlled by the cavity tuner control system to keep $\Delta\psi$ and $V_c = |\tilde{V}_c|$ is controlled by the amplitude feed-back loop and ϕ should fulfill the equation $V_c \cos\phi = U_0$ to compensate the energy loss of the beam. Now we have $\tilde{V}_b$, V_c, ϕ and $\Delta\psi$.

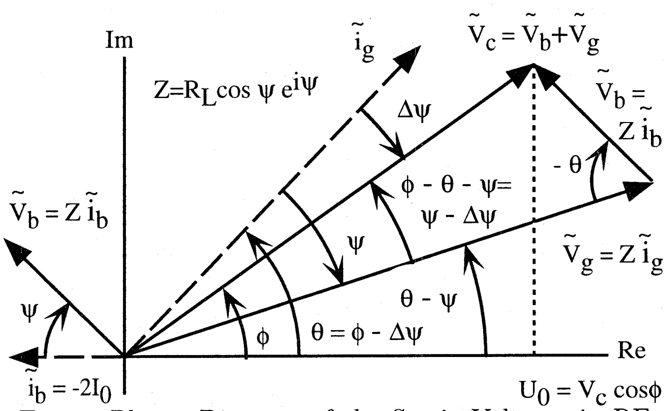

Fig. 1 Phaser Diagram of the Static Voltages in RF Cavity. The positive direction of the angles is counter-clockward.

Glancing Fig. 1, we have

$$|\tilde{V}_b|\sin\theta = |\tilde{V}_c|\sin(\psi - \Delta\psi), \quad \theta = \phi - \Delta\psi \quad (2)$$

and

$$\tan\psi = \tan\Delta\psi - \frac{1}{\cos\Delta\psi}\frac{2I_0 R_L}{V_c}\sin(\phi - \Delta\psi) \quad (3)$$

where we used $|\tilde{V}_b| = |Z\tilde{i}_b| = 2I_0 R_L cos\psi$. The resonant frequency ω_r is obtained through $\tan\psi$ and its definition Eq. (1).

2.2 Feed-Back Controller

Each feed-back loop has one feed-back controllers which consists of a filter circuit and an amplifier of gain G_0. The filter circuit is shown in Fig. 2 and the first stage of the circuit is a high-pass filter and the second is a low-pass filter. The matrix element of the filter for one feed-back loop is

$$Z_f = \frac{1}{i\omega C_L(Z_H + R) + 1}, \quad Z_H = \frac{R_H}{1 + i\omega C_H R_H} \quad (4)$$

where $R = 1k\Omega$ and the other parameters are shown in Table 2.

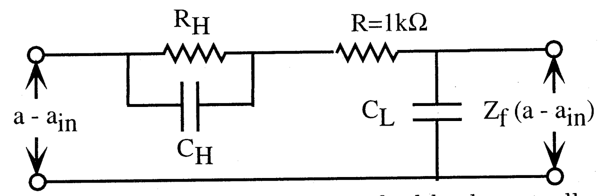

Fig. 2 The filter circuit Z_f in a feed-back controller of the SPring-8 storage ring. The values of R_H, C_H and C_L are listed in Table 2.

Table 2

Filter Parameters. The raw k is for amplitude feed-back loop and the raw ϕ is for phase.

Filter A $C_H = 0\mu F$ $C_L = 24.7\mu F$		$R_H/k\Omega$ at Station		
set		B	C	D
A-1	k	10	10	10
	ϕ	10	10	10
A-2	k	1.1	1.1	1.1
	ϕ	1.1	1.1	1.1
A-3	k	1.1	1.1	1.1
	ϕ	10	10	10
A-4	k	10	10	10
	ϕ	1.1	1.1	1.1

Filter B $C_H = 10nF$ $C_L = 4.7\mu F$		$R_H/k\Omega$ at Station		
set		B	C	D
B-1	k	10	10	10
	ϕ	10	10	10
B-2	k	3	10	10
	ϕ	10	10	10
B-3	k	3	3	10
	ϕ	10	10	10

At the experiment, we used two set of C_H and C_L;Filter A and Filter B, and used several set of value of R_H for six feed-back loops;A-1,2,3,4 for Filter A and B-1,2,3 for Filter B, as shown in Table 2.

The frequency response of the feed-back loop at zero beam current is shown in Fig. 3 and Fig. 4 for Filter A and Filter B with several values of R_H. Filter A has narrower frequency range compared with Filter B because of the large value of C_L.

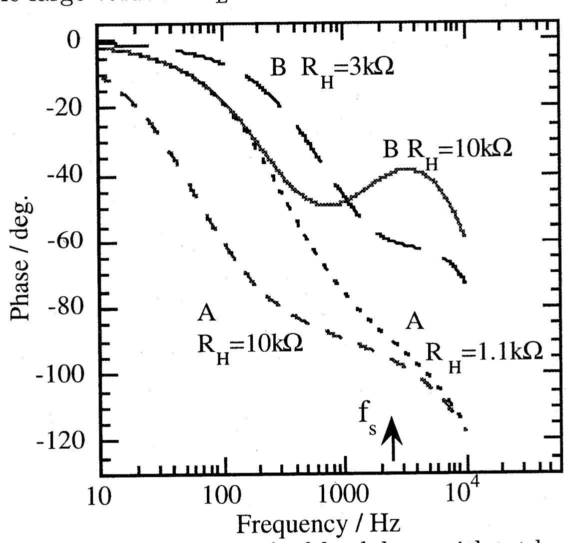

Fig. 3 Phase shift of the feed-back loop without beam current for parameters listed in Table 2.

2.3 Growth Rate and Frequency Shift

We use the eigenvalue equation Eq.() and Eq.() in [1] to obtain the growth rates and the frequency shifts of the instability. At solving these equations, we have to care that C, R, G_0 and Z_f in them have frequency dependence and we used an iteration scheme to solve it.

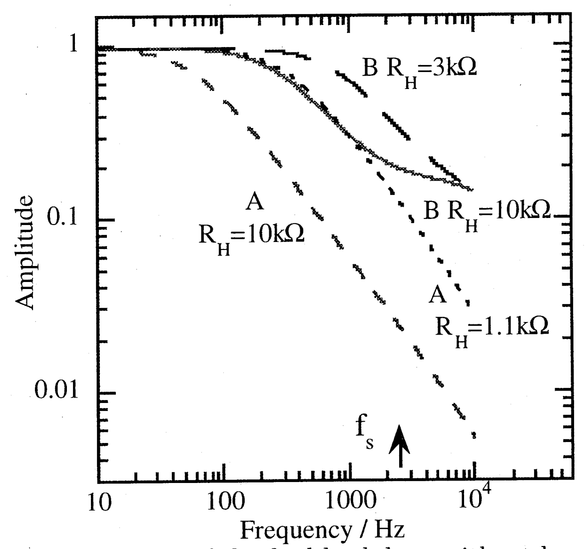

Fig. 4 Loop gain of the feed-back loop without beam current for the parameters listed in Table 2

The calculated growth rates are shown in Fig. 5 and Fig. 6 for parameters in Table 2.

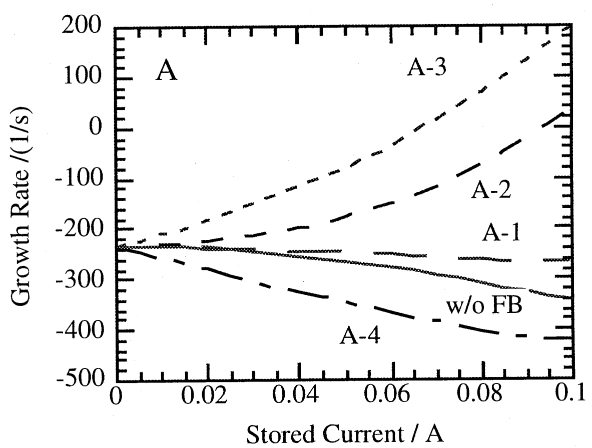

Fig. 5 Growth rate vs stored current for filter A. The growth rate includes radiation damping.

The growth rate is positive above 0.6 A with the parameter B-3.

With the parameter A-3, the gain of amplitude feed-back loop at the synchrotron frequency is larger then A-1 and growth rate is larger than A-1. On the other hand, with the parameter A-4, the gain of phase feed-back loop at the synchrotron frequency is larger then A-1 and growth rate is smaller than A-1.

3 Simple Model

We show a simple model to explain the effect of the feed-back loop on synchrotron motion of a beam.

We assume that Q value of a cavity is lower enough so that the cavity voltage respond immediately to the change of driving current such as i_b or i_g.

Assume the beam executes synchrotron motion. If the beam has a shift of the timing $\tau < 0$ as shown in Fig. 7, this delay produces shift of the beam current, $\Delta\tilde{i}_b$. This produces shift of the cavity voltage, $\Delta\tilde{V}_b$. The feed-back loops try to compensate this shift; the amplitude feed-back loop produces $\Delta\tilde{V}_{gk}$ which reduce energy gain of beam and the phase feed-back loop pro-

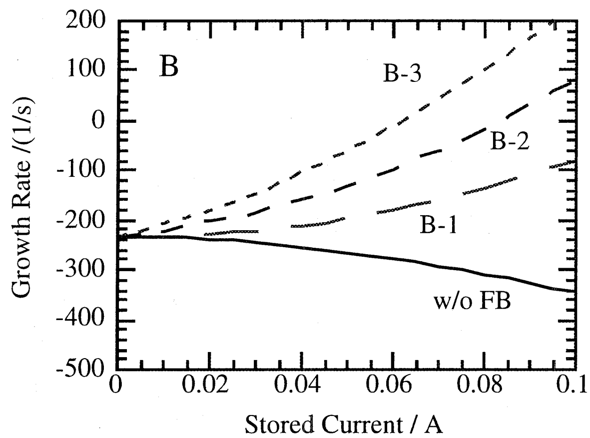

Fig. 6 Growth rate vs stored current for filter B. With B-3, Growth rate exceeds 0 at stored current more than $\sim 0.6A$.

duces $\Delta\tilde{V}_{g\phi}$ which increase energy gain of beam. If the phase delay of the feed-back loop is 0, the total energy gain of the beam during the energy shift $\delta < 0$ in synchrotron motion of the bunch or that during $\delta > 0$ is 0. In actual case, the feed-back loops have a phase delay as shown in thus total energy gain of the beam by $\Delta\tilde{V}_{gk}$ during $\delta < 0$, is negative and the total energy gain during $\delta > 0$ is positive hence the amplitude feed-back loop excite the synchrotron oscillation. On the other hand, $\Delta\tilde{V}_{g\phi}$ increase the energy gain during $\delta < 0$ and decrease the energy gain during $\delta > 0$, which means the phase feed-back loop damps the synchrotron oscillation.

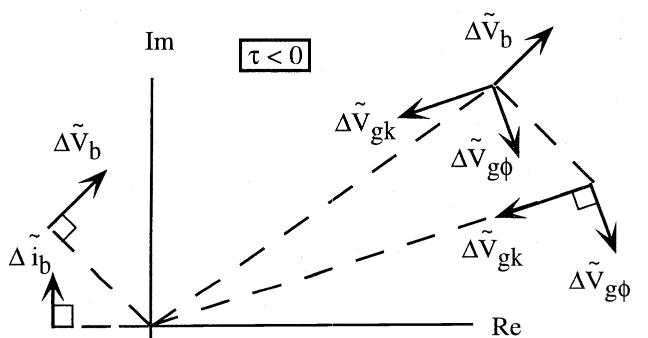

Fig. 7 Phaser diagram of the small shift of the cavity voltage. $\tilde{V}_b$ is produced by small timing delay of the beam for $\tau < 0$. $\tilde{V}_{gk}$ and $\tilde{V}_{g\phi}$ are voltage produced by the amplitude and the phase feed-back loop to $\tilde{V}_b$. The static part of voltages are the same as Fig. 1.

From [1], the amplitude of synchrotron oscillation of the beam, $|\breve{\varphi}|$, is $Q_s = \frac{1}{2}\tau_E\omega_S \sim 20$ times larger than the modulation of the cavity voltage $|\breve{\phi}|$ or $|\breve{k}|$ which drives $\breve{\varphi}$. The loop gain of the feed-back should be smaller than $1/Q_s$ at the worst case that phase delay is 90 deg. But in case of Filter B, the gain at f_s is still higher than $1/Q_s$ and may become unstable.

4 Observation

The experiment was performed with the parameter B-3. The synchrotron motion of the beam was excited by a phase modulation on the RF acceleration voltage and the phase oscillation of the beam current was observed by a phase detector.

The observed frequency response of the synchrotron motion is shown in Fig. 8. As the stored current increases, the peak height become higher and the resonance width become narrower which means damping time is longer because instability growth rate cancel the radiation damping rate. Above stored current 0.6 A, the large amplitude of the synchrotron motion was observed without external excitation.

With A-1, which is the nominal parameter of the storage ring, the growth rate is far below zero and the synchrotron sideband at RF frequency is less than -60dB of main peak.

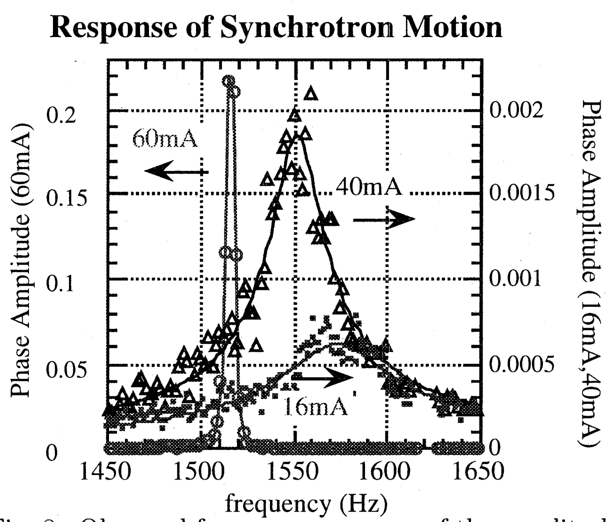

Fig. 8 Observed frequency response of the amplitude of phase oscillation of the beam with Filter B-3. The data for 60mA is 2 magnitude larger than the data at 16mA and 40mA. The damping time/rate of the synchrotron motion from the width of the data are $5.5ms/182s^{-1}$ for 16mA, $12ms/83s^{-1}$ for 40mA and the beam was unstable at 60mA. These values are the expected from the theory as shown in Fig. 6. The peak height is also proportional to the damping time.

5 Conclusion

The effect of the cavity voltage feed-back loop on the Robinson instability was analyzed and growth rate and synchrotron frequency shift is obtained. It shows that ,in actual machine which has slower synchrotron oscillation frequency and large beam loading like the SPring-8 storage ring, the frequency response of the feed-back loop must be slower enough to suppress gain at the synchrotron oscillation frequency to get stable operation at high current.

The author thanks Dr. N. Kumagai, SPring-8, who suggested the existence of the effect of the feed-back loop on the synchrotron oscillation of the beam and encouraged us to search parameters for stable operation.

References

[1] T. Nakamura, "Robinson Instability under Cavity Voltage Feed-back", This proceedings.

[2] P. Wilson, SLAC-PUB-2884(1991),SLAC.