# Study of RF Control System for Superconducting Cavity

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# Abstract

To study the performance of the rf control system in superconducting (SC) linac, we have modeled an SC cavity considering the Lorentz force detuning. The study of an rf control system, which consists of a conventional PID control and feedfoward control, has been carried out with a 600 MHz,  $\beta$ =0.604 cavity. In this study, the field stability less than +/-1% in the rf amplitude and +/-1 deg in the phase was demonstrated by the simulation for the Lorentz force detuning and fluctuation of the beam loading.

## 1. Introduction

The Joint Project between JHP/KEK and NSP/JAERI has been proposed the high intensity proton linac [1]. The linac consists of normal conducting structures and SC structures. For the linac, momentum accuracy ( $\Delta p/p$ ) is required to be less than +/- 0.1 % as the injector for the rapid cycle synchrotron. In turn, the an amplitude and a phase stability of accelerating field should be less than +/- 1 % and +/- 1 deg, respectively.

A feedback control of the SC linac should be applied to stabilize the field for Lorentz force detuning, microphonics and fluctuations of the beam current. In order to control the field for the heavy beam loading, a feedfoward control is necessary because the SC cavity has a long filling time ( $\approx$ sub-millisecond). For pulse operation, the mechanical vibration is caused by the pulsed Lorentz force [2]. The vibration detunes the cavity resonant frequency and disturbs the accelerating field. In the SC cavity with narrow bandwidth, it is one of the significant issues.

The dynamic analysis of the control system including the cavity behavior has been carried out by using a computer simulation code [3]. The analysis allows to predict the performance of the controller with the parameter variation. In this report, we describe the rf system model considering the mechanical vibration and the performance of the controller to stabilize the field.

## 2. Modeling of the SC Cavity

For the cavity with high loaded Q ( $Q_L$ ), the state space equation of the cavity voltage to the beam current ( $I_b$ ) and the generator current ( $I_g$ ) can be represented by two-coupled first order differential equations [4]:

$$\frac{d}{dt} \begin{bmatrix} V_{re} \\ V_{im} \end{bmatrix} = \begin{bmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{bmatrix} \begin{bmatrix} V_{re} \\ V_{im} \end{bmatrix} + \omega_{1/2} \begin{bmatrix} R_L \cdot I_{re} \\ R_L \cdot I_{im} \end{bmatrix}$$
(1)

where  $V_{\rm c} (=V_{\rm re} + j V_{\rm im})$  is the vector cavity voltage, *I* is the vector current  $(I_{\rm b}+I_{\rm g})$ . The band width of the cavity  $(\omega_{1/2})$ , the cavity detuning  $(\Delta\omega)$ , and the equivalent shunt impedance  $(R_{\rm I})$  are given by

$$\omega_{1/2} = \omega_{rf} / 2Q_L$$
, (2),  $R_L = \frac{1}{2} (r / Q) \cdot Q_L$ . (3)

$$\Delta \omega = \omega_0 - \omega_{rf} = 2\pi \cdot (f_{pre} + \Delta f), \qquad (4)$$

The pre-detuning frequency  $(f_{pre})$  is set to minimize the incident rf power during the beam acceleration.

The time varying detuning frequency  $(\Delta f)$  originates in the mechanical vibration. The vibration can be represented as a sum of proper vibrations. For the *k*-th proper vibration mode, the detuning frequency  $(\Delta f_k)$  is given by [5]

$$\Delta f = \sum \Delta f_k \tag{5}$$

$$\frac{d^2 \Delta f_k}{dt^2} + \frac{\omega_{mk}}{Q_{mk}} \cdot \frac{d\Delta f_k}{dt} + \omega_{mk}^2 \cdot \Delta f_k = K_k \cdot \left(\frac{V_c(t)}{V_{c0}}\right)^2 \tag{6}$$

$$K_{k} = \frac{1}{m_{k}} \left[ \left\{ \frac{\delta f}{\delta u} \right\} \cdot \left\{ a_{k} \right\} \right] \cdot \left[ \left\{ a_{k} \right\}^{T} \cdot \left\{ F_{0} \right\} \right]$$
(7)

where  $\omega_{mk}$ : mechanical vibration frequency for the k-th mode,  $Q_{mk}$ : Q-factor of the vibration,  $m_k$ : generalized mass,  $\{F_{\theta}\}$ : Lorentz force vector at the designed cavity voltage of  $V_{c0}$ ,  $\{a_k\}$ : eigenvector for k-th mode,  $\{\partial f / \partial u\}$ : frequency sensitivity vector for cavity wall displacement. The inner product of  $\{\delta f | \delta u\} \cdot \{a_k\}$  and  $\{a_k\}^T \cdot \{F_0\}$  mean the frequency sensitivity for the k-th mode and coefficient with respect to the Lorntz force for the k-th mode, respectively. Those mechanical parameters are obtained by structural ,( $\omega_{mk}$ ,  $Q_m$ ,  $m_k$ ,  $\{a_k\}$ ) and electromagnetic  $(\{\delta f | \delta u\}, \{F_0\})$  analysis utilizing the simulation codes "ABAQUS" and "SUPERFISH".

# 3. Simulation Model of rf System

The rf system simulation makes use of the programming language MATLAB/Simulink. The MATLAB is the interactive software that offers numerical calculation, data analysis and program function. The Simulink linked to the MATLAB is the block diagram simulator with graphical user interface. The software is available at most controller design fields.

In the Simulink, the state space equations are expressed by block diagrams. For an example, the detuning frequency equation is shown in Figure 1. The block diagram of the rf



Fig. 1 Block diagram of the Simulink to solve the vibration equation.

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Fig. 2 Schematic block diagram of the rf system

system is shown in Fig.2. In this study, we choose the rf control scheme that is the conventional I/Q control and PID feedback system with the delay time. The  $V_c$  set-point timetable is set as information of the designed operation-pattern of  $V_c$ . The feedfoward timetable is set as the predictable operational patter of  $V_g$ , which is applied to assist the feedback control for the beam loading and Lorentz detuning. The Output of the feedback and feedfoward controller is limited within 1.14 times of the nominal generator voltage (= $2V_c$ ) at the limiter. The beam voltage ( $2R_L \cdot I_b$ ) can be modeled with arbitrary time varying amplitude and phase as specified in the Beam block.

#### 4. Simulation Results

## 4.1 Simulation Parameters

The simulation of the rf control system is carried out for the SC cavity that is designed at an rf frequency of 600 MHz, geometrical  $\beta$  of 0.604 and wall thickness of 3 mm. The rf characteristics are shown in Table 1. The  $Q_L$  is the value in the optimum coupling ( $\beta_{opt}$ ). The  $\beta_{opt}$  is given by the I<sub>b</sub> of 28mA (50 mA peak current x 0.56 intermediate pulse duty) and the average synchronous angle ( $\phi_s$ ) of -30 deg. The mechanical characteristics are shown in Table 2. In this study, we take into account the proper vibrations up to 4-th mode to reduce task of the computer.

From the Bode diagram of the cavity transfer function, we take the three-parameters of the PID controller which consist of proportional control gain  $(K_p)$ , integral control time constant  $(T_1)$  and differential control time constant  $(T_D)$  for the controller delay time (L). Applying Laplace-transformation and solving for equation (1) yields a transfer function,

$$\begin{bmatrix} V_{re.s} \\ V_{im.s} \end{bmatrix} = \frac{\omega_{1/2}}{(s + \omega_{1/2})^2 + \Delta \omega^2} \begin{bmatrix} s + \omega_{1/2} & -\Delta \omega \\ \Delta \omega & s + \omega_{1/2} \end{bmatrix} \begin{bmatrix} R_L I_{re.s} \\ R_L I_{im.s} \end{bmatrix}$$
$$= \begin{bmatrix} Hc & -Hs \\ Hs & Hc \end{bmatrix} \begin{bmatrix} R_L \cdot I_{re.s} \\ R_L \cdot I_{im.s} \end{bmatrix}.$$
(8)

The off-diagonal elements (Hs) are the coupling factor from real to imaginary parts. Neglecting the contribution of the Hs for voltage (V(s)) to simplify the parameter derivation, the transfer function becomes the one-input one-output system. Figure 3 shows the block diagram of the closed loop adding the controller delay. In this system, stability criterion point is where the phase of the closed loop transfer function reaches -180 degree. At that frequency, the negative feedback turns into positive feedback. (If the loop gain is larger than 1 (0 dB) at this point, the loop oscillates.) Figure 4 shows the frequency responses of the loop for the several delay time from 0 to 20  $\mu$ s. According to an experimental role[6], the optimum PID parameters can be

Table 1 rf characteristics of the cavity. (f=600MHz,  $\beta$ =0.604, thickness = 3mm)

1-600 MHz, $p-0.6$	504, thickness $= 3$ m
$Q_0$ .	1.11E+10
$Q_{ m L}$	1.08E+06
r/Q	115.39 (Ω/cav.)
$V_{\rm c}$	3.01 (MV)
Filling Time $(\tau)$	0.571 (ms)

Table 2 Mechanical characteristics of the cavity					
Mode Number k	1	2	3	4	
$\omega_{mk}$ (k rad/s)	3.63	10.0	15.4	21.6	
$m_{\rm k}$ (kg)	1.34	1.53	1.71	1.55	
$\mathcal{Q}_{mk}$	100	100	100	100	
$\{a_k\}\{F_0\}$ (N)	3.29	-7.28	-6.18	0.98	
$\{df/du\}\{a_k\}$ (MHz/m)	-436	2150	1820	-64.5	



Fig. 3 Block diagram of the closed loop



given by  $K_p=0.6K_c$ ,  $T_1=0.5/f_c$  and  $T_D=0.125/f_c$  when the gain and frequency at the critical point are  $K_c$  and  $f_c$ , respectively. Those parameters are summarized in Table 3.

### 4.2 Simulation Results

We estimate the influence of the Lorentz detuning for the rf system. In this simulation, the controller delay time is 20µs and the initial detuning frequency is zero. The behavior of the rf control system in single pulse is shown in Fig 5. The vector voltages are converted from I/Q components on the simulation into amplitude and phase components (Fig5 (a), (b)). In the designed  $V_c$  waveform written in the  $V_c$  set-point timetable, the amplitude gradually rises for 1ms and keeps the flattop for 1 ms. The phase ( $\phi_c$ )

Table 3 PID parameters of the rf system for the delay time

Delay Time	K <sub>c</sub>	$T_{\rm c}$	$\overline{K_{p}}$	$T_{\rm I}$	$T_{\rm D}$
μs	dB	μs	dB	μs	μs
5	45	20	41	10	2.5
10	39	40	35	20	5
20	33	83	29	42	10

maintains at -30 degree. On the decision of the set-point timetable, it is possible to set the fast field filling less than 1ms, but the mechanical vibration will be enhanced. The beam voltage is applied at an ideal waveform  $(V_{\rm b0})$  predicted in the feedfoward timetable.

Fig. 5(c) shows the detuning frequency. The detuning frequency varies to -200 Hz in the pulse and the frequency change of 46 Hz arises during the beam pulse. The frequency change corresponds to the cavity phase of about 5 degree. The influence of the cavity phase change is compensated by the control of  $\phi_g$ . (Actually, the generator voltage ratio of I to Q component is controlled.) The field errors ( $\varepsilon_A$ ,  $\varepsilon_{\phi}$ ) are less than 0.1% and 0.1 degree for the beam acceleration (shown in Fig. 5 (d)). In this simulation, the influence of the Lorentz detuning is sufficiently compensated by the rf controller.

Applying the larger amplitude of the beam voltage (1.2 times of  $I_{b0}$ ), the performance of the rf control system is examined. Figure 6 shows the simulation result in the scope of the beam pulse. The feedback acts 20µs after applied the  $V_b$ . During this period the  $V_c$  decreases becouse the  $V_g$  is insufficient to compensate for the  $V_b$ . Maximum  $\varepsilon_A$  of -0.76 % and  $\varepsilon_{\phi}$  of 0.25 degree are observed. For several delay times the simulations are done. Table 4 lists the results of the maximum  $\varepsilon_A$  and  $\varepsilon_{\phi}$ . In case of the rf system with the shorter delay time, the PID parameters can be set as the larger gain and the smaller time constant, and the feedfoward begins to work at early time. In consequence, the field error can be suppressed. The feedback control response of several µs is required for influence of beam loading error of +20 %..

## 5. Conclusion

We have modeled the SC cavity considering the cavity mechanical vibration that is excited by the pulsed Lorentz force. The rf system model is constructed on the MATLAB/Simulink. The simulation has been carried out for the cavity that is designed at 600 MHz,  $\beta$ =0.604 and wall thickness of 3mm. In the I/Q control model, the PID feedback and feedfoward controller is utilized. The optimum PID parameter is calculated for the several controller delay times. In the estimation of the influence of the Lorentz detuning, the field stability of less than +/-1 %, +/-1degree is achieved by the rf control with the delay time of 20 µs. The feedback control response of about µs order is necessary to compensate the cavity voltage for the beam loading fluctuation of +/- 20% order.

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Fig 5 Simulation result for the rf control system.



Fig 6 Amplitudes and errors for the beam current error.

Table 4 Maximum field error for the control delay time.Delay time (us)20105

Delay time (µs)	20	10	5
$\varepsilon_{\rm A} \max(\%)$	-0.76	-0.39	-0.20
$\varepsilon_{\phi} \max (\text{deg})$	0.25	0.13	0.065

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