A Supermode Analysis of Michelson Resonator FELs

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Abstract

Evolution of laser field in a Michelson resonator FEL is studied via supermode analysis. It is found that the lasing dynamics in a Michelson FEL can be described by three dimensionless variables: cavity loss, equivalent cavity detuning and the interference parameter. Small-signal gain and temporal profile of laser pulse can be obtained as eigenvalue and eigenstates of the supermode equation.

1 Introduction

Evolution of laser filed in a short-pulse free-electron laser (FEL) oscillator shows various behavior, stable, limit cycle and chaotic lasing, according to resonator parameters: loss and detuning [1]. Controlling the lasing behavior is, therefore, a key issue to realize wide application of FELs in scientific and industrial fields.

Michelson FEL was first developed to increase spectral purity of FEL by establishing phase coherence between successive optical pulses in the resonator, where the secondary arm provides extra delay of one rf period [2].

Numerical analyses recently showed that a Michelson resonator also has ability to suppress the emergence of chaos even with parameters which gives chaotic lasing in a conventional two-mirror resonator FEL [3]. The path difference between two arms of the Michelson resonator for the chaos suppression is chosen so as to provide small delay shorter than the electron bunch. The mechanism of chaos suppression might be considered as a result of frequency filtering or suppressed superradiant, the detail has not been cleared however.

In the present paper lasing dynamics of a Michelson FEL is studied via supermode analysis which gives intuitive understanding of lasing dynamics in a Michelson FEL and allows further investigation of the chaos suppression.

In section 2, we introduce lasing dynamics in a Michelson FEL. An eigenmode equation which gives supermodes in a Michelson FEL is derived in section 3.

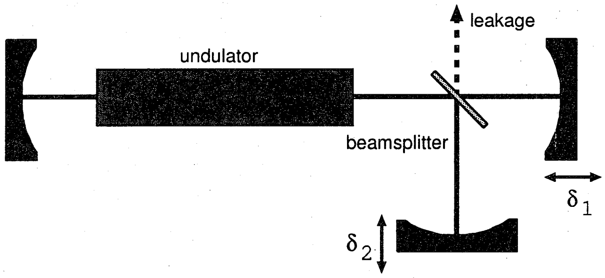

Fig. 1 Michelson resonator FEL

2 Lasing Dynamics in Michelson FEL

We start with one-dimensional FEL equations described in terms of scaled dimensionless variables and use the same notation as ref.[1].

Let normalized cavity detuning for primary and secondary arms of a Michelson resonator be δ_1 and δ_2 respectively, and we define positive sign of δ for cavity shortening from perfect synchronism of electron bunch interval. The laser pulse is reflected by mirrors of the cavity and evolves through successive interaction with electron bunches every round trip, where the laser pulse pushed forward or backward relative to the electron bunch according to the cavity detuning from the synchronism.

The input field for the (n+1) pass is given as

$$A_0^{(n+1)}(\xi-\delta_1-\delta_2) = rt_s^2 A_f^{(n)}(\xi-\delta_2) + rr_s^2 A_f^{(n)}(\xi-\delta_1) \quad , \qquad (1)$$

where A is the slowly varying complex amplitude of the radiation in dimensionless form, A_0 and A_f are field at the entrance and the end of undulator, respectively, $\xi = (ct-z)/\Delta$ is longitudinal coordinate in the moving frame normalized by slippage distance $\Delta = \lambda N_w$, t_s^2 and r_s^2 are transmittance and reflectance of the beam splitter in the Michelson resonator and r represents energy loss at mirrors.

An ideal beam splitter has no energy loss : $t_s^2 + r_s^2 = 1$. The beam splitter, however, still allows leakage of optical power due to phase mismatch between radiations reflected from two arms, that is frequency filtering, the intrinsic effect of a interferometer.

We limit the analysis to small single-pass gain and assume both cavity detuning and loss are small, and hence Eq.(1) can be expanded in Taylor series and rewritten with the FEL equations:

$$\frac{\partial A(\xi,\tau)}{\partial\tau} - \nu^* \frac{\partial A(\xi,\tau)}{\partial\xi} + \frac{\alpha}{2}A(\xi,\tau) = \eta\left\langle \exp[-i\tilde{\theta}(\xi,\tau)]\right\rangle - \rho^* \frac{\partial}{\partial\xi}\eta\left\langle \exp[-i\tilde{\theta}(\xi,\tau)]\right\rangle \quad , \qquad (2)$$

$$\frac{\partial^2\tilde{\theta}(\xi,\tau)}{\partial\xi^2} = -\left\{A(\xi,\tau)\exp[i\tilde{\theta}(\xi,\tau)] + c.c.\right\} \quad , \qquad (3)$$

where short bunch limit is assumed and parameters are normalized by small-signal gain γ following ref. [1]: $\tau = \gamma n$, $\alpha = \alpha_0/\gamma$, $\alpha_0/2 = 1-r$, $\nu^* = (\delta_1+\delta_2)/\gamma - \rho^*$, $\rho^* = r\left(t_s^2\delta_2 + r_s^2\delta_1\right)/\gamma$.

Equation (2) shows that lasing dynamics in a Michelson FEL is determined by three dimensionless parameters: scaled cavity loss α, equivalent cavity detuning ν^* and the interference parameter ρ^*.

The second term in the right-hand side of Eq.(2) is proportional to the derivative of bunching term along ξ -axis, which will vanish, if the magnitude of bunching is uniform along ξ -axis, that is the case no phase variation exists in driving term. The second term, therefore, represents the interferometric property of a Michelson resonator and never appears in FELs with conventional two-mirror resonators.

The interference parameter in the definition of the equivalent cavity detuning reduces the mirror detuning δ_1 by factor $(1-rr_s^2)$. This relaxation of effective detuning results in the broadening of cavity detuning curve, which is one of the most important effects in a Michelson FEL [2].

It is also found that the values of ν^* and ρ^* are restricted by δ_1 and δ_2. Since the cavity mirror and the beam splitter have reflectance of $0 \le r \le 1$ and $0 \le r_s^2 + t_s^2 \le 1$, the interference parameter may have the value

$$\min\{\delta_1, \delta_2\} \le \gamma\rho^* \le \max\{\delta_1, \delta_2\} \ . \tag{4}$$

The equivalent detuning is also allowed to have value:

$$\min\{\delta_1, \delta_2\} \le \gamma\nu^* \le \max\{\delta_1, \delta_2\} \ . \tag{5}$$

3 Supermode Analysis of Michelson FEL

The FEL equations expressed in terms of collective variables can be rewritten in a form including the term representing the interferometric property of a Michelson resonator:

$$\frac{\partial A}{\partial \tau} - \nu^* \frac{\partial A}{\partial \xi} + \frac{\alpha}{2} A = B - \rho^* \frac{\partial B}{\partial \xi} \ , \tag{6}$$

$$\frac{\partial B}{\partial \xi} = -iP \ , \tag{7}$$

$$\frac{\partial P}{\partial \xi} = -A \ , \tag{8}$$

where $B = \left\langle \exp(-i\tilde{\theta}) \right\rangle$ and $P = \left\langle \tilde{p} \exp(-i\tilde{\theta}) \right\rangle$ are the collective variables called as the bunching parameter and the phase-momentum average, respectively.

Equations (6) – (8) have an eigenstate:

$$A(\xi,\tau) = \exp\left[(\mu - \alpha/2)\tau\right] \Phi(\xi) \ , \tag{9}$$

$$B(\xi,\tau) = \exp\left[(\mu - \alpha/2)\tau\right] \Psi(\xi) \ , \tag{10}$$

$$P(\xi,\tau) = \exp\left[(\mu - \alpha/2)\tau\right] \Gamma(\xi) \ . \tag{11}$$

Substituting Eq.(9)–(11) into Eqs.(6)–(8), we find

$$\nu^* \Phi''' - \mu \Phi'' - i\rho^* \Phi' + i\Phi = 0 \ , \tag{12}$$

which is similar to one derived in ref.[1] but includes the ρ^* term resulting from the interference in a Michelson resonator. Once an eigenvalue μ is found, single-pass gain can be obtained as $\mathcal{G} = 2\gamma Re(\mu)$ and linear phase shift is given as $\phi = Im(\mu)$. The eigenfunction $\Phi(\xi)$ represents a supermode which describes the evolution of temporal profile of laser pulse.

Solutions of Eq.(12) depends on the boundary condition for $\Phi(\xi)$ which is divided into three cases as follows:

(1) $\delta_1 < 0$ and $\delta_2 < 0$
The laser pulse is pushed backward every round trip and $\Phi(0) = 0$ is given as the boundary condition. No nonzero solutions for $A(\xi,\tau)$ exist in this case as well as cavity lengthening in conventional two-mirror resonator FELs.

(2) $\delta_1 > 0$ and $\delta_2 > 0$
The laser pulse is pushed forward every round trip and $\Phi(1) = 0$ is given as the boundary condition. Nonzero solutions for $A(\xi,\tau)$, that is supermodes having positive gain, can be obtained with the algorithm described in ref. [1]. Figure 2 shows small-signal gain of the first two supermodes as a function of equivalent cavity detuning, where the interference parameter is chosen as $\rho^* = 0.01$ and $\rho^* = 0.1$. It can be seen that the supermode gain curves of Michelson FELs part from the gain curve of conventional FELs with increasing of ρ^*.

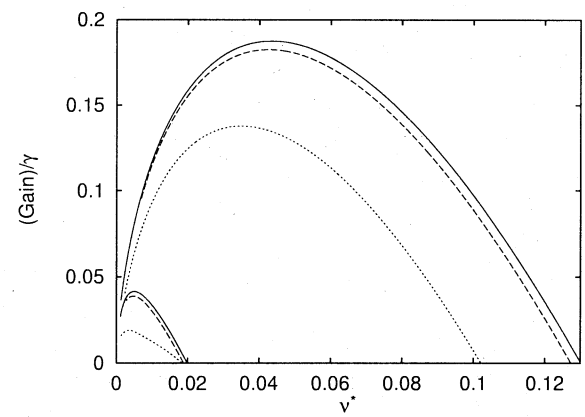

Fig. 2 Small-signal gain of the first two supermodes in Michelson FEL ($\delta_1 > 0$, $\delta_2 > 0$) vs equivalent detuning: $\rho^* = 0.01$ (dashed line), $\rho^* = 0.1$ (dotted line), conventional resonators (continuous line).

(3) $\delta_1 \, \delta_2 < 0$
This is the case chaos suppression occurs [3]. A part of laser pulse is pushed forward and the rest is pushed backward every round trip. The boundary condition is not given in a form as $\Phi(\xi) = 0$ and Eq.(12) cannot be solved analytically. The profile of laser filed and small-signal gain, therefore, are calculated numerically with a pulse propagation code [3] instead of finding analytical solutions.

Calculated small-signal gain for Michelson FELs with cavity detuning as $\delta_1\delta_2 < 0$ is plotted in Fig.3. The leakage from beam splitter is also shown in Fig.4. It shows that a Michelson FEL with cavity detuning as $\delta_1\delta_2 < 0$ has gain curve similar to the first supermode of conventional FELs. The gain, however, does not reduce as fast as the supermode with approaching the equivalent detuning to zero.

Figure 5 shows the profile of saturated laser filed in Michelson FEL with parameters $\delta_1\delta_2 < 0$, $\rho^* = 0.01$,

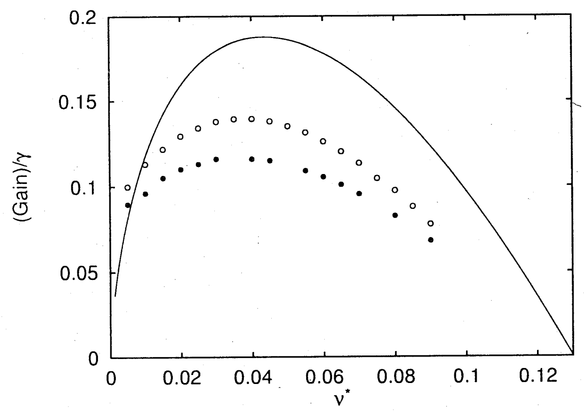

Fig. 3 Small signal gain for Michelson FEL ($\delta_1\delta_2 < 0$) vs equivalent cavity detuning: $\rho^* = 0.01$ (∘), $\rho^* = 0.1$ (•), the first supermode for FEL with conventional resonators (solid line).

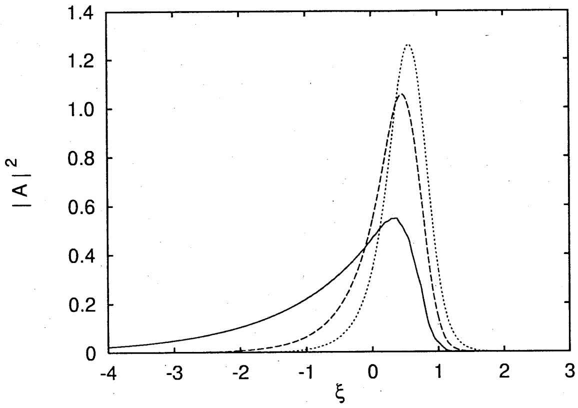

Fig. 5 Laser field profile for Michelson FEL with parameters: $\rho^* = 0.01$, $\alpha = 0.05$, $\nu^* = 0.075$ (continuous line), $\nu^* = 0.025$ (dashed line), $\nu^* = 0.01$ (dotted line).

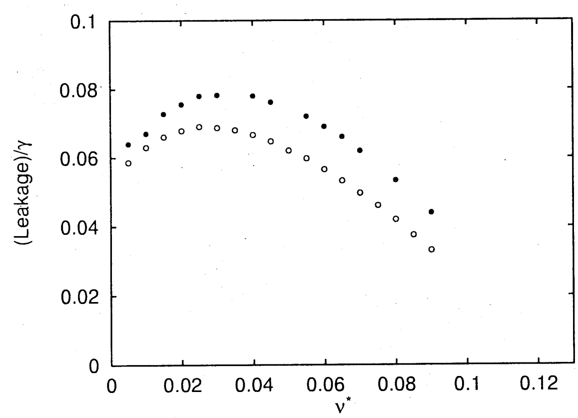

Fig. 4 Normalized leakage power from the beam splitter of Michelson FEL: $\rho^* = 0.01$ (∘), $\rho^* = 0.1$ (•)

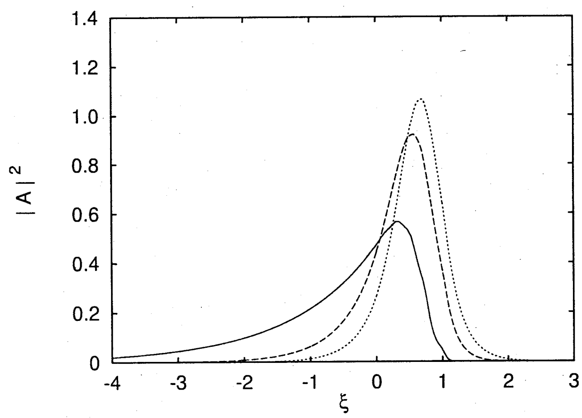

Fig. 6 Laser field profile for Michelson FEL with parameters: $\rho^* = 0.1$, $\alpha = 0.05$, $\nu^* = 0.075$ (continuous line), $\nu^* = 0.025$ (dashed line), $\nu^* = 0.01$ (dotted line).

$\alpha = 0.05$ and $0.01 < \nu^* < 0.075$. Results for $\rho^* = 0.1$ are also shown in Fig.6. It shows that laser field penetrates into the region $\xi > 1$ where no laser filed exists in case of FEL oscillators with conventional two-mirror resonators. The penetration due to radiation field pushed backward by cavity lengthening becomes more remarkable with decreasing of ν^* and increasing of ρ^*.

4 Summary

Analytical study on Michelson FEL have been made and it is found that lasing dynamics of Michelson FEL can be described by three dimensionless parameters: cavity loss, equivalent cavity detuning and the interference parameter.

We have derived the supermode equation for a Michelson FEL which gives small-signal gain and evolution of temporal profile of laser pulse. Laser field profile and small-signal gain in Michelson FEL is also obtained numerically for various parameters.

References

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