Novel X-ray Source Using Collisions of Circulating Relativistic Electrons and a Wire Target

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Abstract

The novel method for generating brilliant x-ray beams is proposed, in which inelastic collisions of circulating relativistic electrons and a thin wire target are used. High brilliance of this new photon source stands on narrow angular divergence due to the kinematics of relativistic electrons, and repeatedly use of electron beams. The estimated brilliance of this source based on a 50 MeV electron storage ring is comparable to that of compact synchrotron light sources.

1. Introduction

X-ray emissions are generated when electrons are decelerated by either an electric or a magnetic force. classical x-ray tubes and rotating anode sources use atomic electrical forces, and synchrotron radiation (SR) uses bending magnets or wigglers. All of these sources generate continuous and incoherent spectra. The SR emissions are brighter than x-ray tube emissions only because SR emissions have a narrower angular divergence, but the integrated total radiated power is almost same for both methods. Note that the divergence is determined by the kinematics of the electron beam regardless of what the decelerating force is. The divergence is about $1/\gamma$ in either cases, where γ is the Lorentz factor. Using relativistic electron beams is the key issue for achieving the narrow angular divergence of x-ray beams.

The bremsstrahlung spectrum extends up to the incident electron energy. This implies that 100 keV electron beam is enough to produce 100 keV x-rays. X-rays are therefore produced more efficiently with conventional x-ray tubes than with SR.

The above consideration leads to a simple idea for a novel xray source that uses collisions of circulating high-energy electrons and solid targets in a storage ring. The narrow angular divergence is secured by the high electron energy. High energy x-rays can be easily generated by the bremsstrahlung due to the inelastic collisions inherent with this kind of source.

Using a thin target is also the key issue for present x-ray source in order to avoid increasing angular divergence of x-ray beam due to multiple scatterings. The electron beam passing through the thin target is utilized repeatedly so that the x-ray productivity will be significantly enhanced in comparison with the case using a linac and a thick target. A thin target is useful to minimize absorption of x-rays and electrons which cause heating problem of the target. The continuous injection of an electron beam into the storage ring at full energy is a way to compensate for the lost beam and to keep a high, constant beam current. We gain yet another advantage when a thin wire target is used, because effective x-ray source size is determined by the width of this wire.

2. Bremsstrahlung yield

As well known the differential cross section of bremsstrahlung has been quantum-mechanically calculated by Bethe and Heitler¹, Shiff², and others in the 1950's. A summary of theories and experiments is given by Koch and Motz.³. In order to evaluate the bremsstrahlung yield in comparison with synchrotron light, we define here the brilliance of the bremsstrahlung as:

$$\frac{d^{2}N_{Br}}{d\Omega dS} = \frac{I}{1.6E - 19} \frac{\alpha_{t}}{S_{b}} \cdot n_{t} \cdot \chi_{t} \frac{4Z(Z + 1)r_{0}^{2}}{137}$$

$$\frac{dk}{k} \frac{1}{\chi_{a}\chi_{y}\pi\theta_{Br}^{2}} \int_{0}^{\theta_{Br}} \sin(\theta)d\theta$$

$$\left\{ \frac{16\Theta^{2}E_{o}^{2}}{(1+\Theta^{2})^{4}} - \frac{(E_{o} + E)^{2}E_{o}}{E(1+\Theta^{2})^{2}} + 2\ln(\frac{EE_{o}}{k}) \left[\frac{(E_{o} + E)^{2}E_{o}}{E(1+\Theta^{2})^{2}} - \frac{4\Theta^{2}E_{o}^{2}}{(1+\Theta^{2})^{4}} \right] \right\}$$

$$----- (1)$$

where *I* is the beam current, n_t and χ_t are the target density and thickness, k, E_0 , and *E* are the x-ray energy, the incident electron energy, and the scattered electron energy measured in the electron rest of mass energy units, respectively. The ϑ is defined as $\vartheta = \theta \cdot E_o$, where θ is the angle between the directions of photons and incident electrons. The integral in eq.(1) is carried out over $0 - \theta_{Br}$. We assume $\theta_{Br} = 1/\gamma$ which covers the peak in the angular distribution around $0.5/\gamma$. Thus, this definition gives averaged brilliance, but not the peak value. The brilliance is a normalized value with the source size $\chi_x \cdot \chi_y$ where $\chi_x \chi_y$ are the horizontal and vertical target size or beam size, whichever is smaller. While the target size is smaller than the beam size, the brilliance is determined by the target size. Please note that in eq. (1) we take the effective beam current, $(I.*\alpha_t/S_b)$, where α_t is the x-ray source area which is determined by the target size, and S_b the beam size.

3. Beam current

In the next we estimate the obtainable ring beam current resulting from collisions of the beam and the target material with a continuous beam injection. The time evolution of the stored beam current (dI/dt) may be expressed by the following equation:

$$\frac{dI}{dt} = R_i \cdot \varepsilon_i \cdot I_0 \cdot \delta t \cdot f - \mu I - \eta I^2.$$
 (2)

The first term of the right hand side is the beam growth rate by injection, which is given by multiples of the injection rate, R_i , the total injection efficiency, ε_i peak current, Io, the duration in which the beam is accepted by the storage ring, δt , and the frequency of the circulating beam, f. The current decay rate can be represented by first- and second-order terms of I with the coefficients μ and η . The meanings of these rate coefficients will be clarified in the following.

The stored beam decays by two principal effects. One effect is Touschek scattering which is large-angle scattering caused by Coulomb repulsion between electrons in a bunch. The other effect is electron-target scattering as the formula is known by the problem of gas scattering in a storage ring. In electron-target scattering both inelastic and elastic scattering on target nuclei occur, with inelastic scattering causing bremsstrahlung.

Elastic scattering on nuclei of the target leads to angular kick for the betatron motion of the electron beam. If the induced betatron amplitude exceeds the transverse acceptance of the ring, A- $_{c}$, the beam will be lost. The total cross section for this process is expressed by:⁴

$$\sigma_{el} = \frac{2\pi r_e^2 Z^2}{\gamma^2} \cdot \frac{\langle \beta \rangle}{A_c},$$
(3)

where Z is the atomic number, r_e is the classical electron radius, and $\langle \beta \rangle$ is either the horizontal or vertical betatron amplitude, whichever limits the beam circulation. The transverse acceptance is actually limited by either the half-chamber aperture or the dynamic aperture, b, at the place where the betatron amplitude is β $_o$. The acceptance is then given as $A_c = b^2/\beta_o$, and $\langle \beta \rangle = \beta_o$. Note that this cross section decreases quadratically with increasing electron energy.

Bremsstrahlung is an inelastic scattering process that leads to an energy loss for the circulating electron. The electron will be lost

Table 1. Machine parameters of a 50 MeV electron storage ring
and Brernsstrahlung intensities produced by the scattering of the
stored beam and targets are shown. Photon intensities are
calculated for 100 keV x-ray.

Machine parameters			
Electron energy [MeV]		50	
Orbit radius [m]		0.15	
SR loss/turn, electron [eV]		3.69	
n-value		0.01	
RF voltage [keV]		120.00	
Hamonics		8	· · ·
RF frequency		2.45E+09	
Horizontal damping rate		50	
[/sec]			
Touschek half life [sec] at		60	
10A			
Horizontal beam		9	
width[mm]			
Vertical beam width[mm]		0.5	
Bunch length[mm]		9	
Bremsstrahlung	C-foil	C-wire	W-wire
target density [number/m ³]	1.13E+29	1.13E+29	6.25E+28
Target width [µm]		100	10
Target thickness [µm]	10	100	10
Cross section [m ²]	9.73E-30	9.73E-30	1.29E-27
Radiation loss/electron [eV]	1.07E+03	1.07E+04	7.87E+04
Beam loss rate by inelastic	7.64E+03	2.03E+03	1.25E+03
scat. [/sec]			
Beam loss rate by elastic scat.	2.65E+04	7.04E+03	5.16E+03
[/sec]		·.	
Injector parameters			
Injection rate [Hz]		50	
Peak current [A]		10	
Injection efficiency		6.0E-08	
Integrated beam current [A]	0.379	1.43	2.02
Photon flux for 100keV xray	5.91E+10	5.91E+10	6.13E+10
Brilliance for 100keV x-ray	1.79E+09	5.06E+10	5.25E+11

if the energy loss exceeds the limiting longitudinal momentum halfaperture, $(\Delta p / p)_{\text{max}}$ of the ring that is proportional to the radiofrequency bucket height. The total cross section for this inelastic process leading to the beam loss is given by:⁵

$$\sigma_{Br} = \frac{4Z^2 r_e^2}{137} \frac{4}{3} \left[\ln(\frac{183}{Z^{1/3}}) \right] \left[\ln(\frac{1}{(\Delta p/p)_{\text{max}}} - \frac{5}{8} \right].$$
(4)

Note that this cross section is independent of the electron energy.

The total beam loss rate, μ , due to the above elastic and inelastic scattering can be expressed as:

$$\mu = -\frac{1}{I}\frac{dI}{dt} = (\sigma_{el} + \sigma_{Br}) \cdot n_t \cdot \chi_t \cdot f_{RF} \frac{\alpha_t}{S_b}, \qquad (5)$$

where n_t and χ_t is the target density and thickness, respectively. The α_t is the x-ray source area which is determined by the target size when it is smaller than the beam size, Sb. If the target size is larger than the beam size, $\boldsymbol{\alpha}_t$ is equal to the beam size. The calculated beam loss rates for a 50 MeV ring with various target materials and shapes are shown in Table. 1. A half-momentum aperture of $(\Delta p/p)_{\text{max}}=0.06$ and the vertical half-chamber aperture of b=3 cm is assumed in these calculation. A 10 µm-thick carbon foil gives beam loss rate of 7640/sec for the inelastic and 26500/sec for the elastic scattering. In the case of 10µm¢ tungsten wire, these are 1250/sec and 5160/sec, respectively on assumption of RMS beam size of 9 x 0.5mm. Increasing beam size simply leads to reducing beam loss rate, and consequently increasing ring current according to eq. (2) and (5). The maximum beam current is reached when the beam injection and the beam loss are balanced. Then the maximum beam current is obtained by solving eq. (2) for dI/dt=0.

We summarize the resulting maximum beam current and the machine parameters of a 50 MeV electron storage ring in Table 1. In this calculation we assumed 50A peak current and 10 Hz injection rate. We assumed an injection efficiency of ε_i =0.6 which is conservative for a resonance injection method.⁶ We assumed a 10 µm thick cabon foil, 100 µm carbon wire, and 10µm tangsten wire taget, but the thickness is unimportant for estimating the maximum x-ray yield, because the effect of the target thickness and density are canceled out in this calculation. The x-ray cross section is proportional to the target thickness and density, but the maximum beam current is inversely proportional to these target parameters. The x-ray yield is the multiple of the cross section and the beam current. The width is important for higer brilliance. The beam size is unimportant for the total x-ray yield. It may growth due to the scattering, but the total x-ray yield will be unchanged. Consequently the maximum obtainable x-ray production is determined by the maximum output power of the injector.

4. Conclusion

In summary, we believe that the use of inelastic collisions of relativistic electron and of solid targets is a promising new way to generate brilliant photon beams ranging from soft x-ray to beyond 100 keV x-ray. The size of this novel source must be very attractive compared with compact SR sources.

It is worth to point out that if the size of the ring is not problem, the higher energy electron storage ring is advantageous. For instance a 300MeV ring generates thousands times brighter xray beam, since the angular distribution is less than 1/6, the beam loss rate by elastic scattering is reduced 10 times, and the beam size is reduced 100 times.

In this paper we didn't include other radiation mechanisms like transition and Cerenkov radiations. It is known that the strength of transition radiation may reach the same level as bremsstrahlung, and that photon energy from 50 MeV electrons is available up to a few keV. It is also known that stacked foils generate coherent x-ray beam. We therefore expect that the new xray source will produce much more power than we have described above.

References

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