

## Analysis of a Three-Cell Cavity which Suppresses Instabilities Associated with the Accelerating Mode

Y. Yamazaki and T. Kageyama  
KEK, National Laboratory for High Energy Physics  
1-1 Oho, Tsukuba-shi, Ibaraki-ken 305, Japan

### Abstract

In a large ring with extremely heavy beam loading such as a B-factory it is possible that the accelerating mode, itself, gives rise to a longitudinal coupled-bunch instability. In order to solve this problem Shintake proposed to attach a storage cavity to an accelerating cavity. The present paper shows that the system can be put into practical use, if one adds a coupling cavity in between the two cavities.

### I. INTRODUCTION

In a B factory an extremely high beam current of an order of a few mA must be stored in order to obtain the required high luminosity. The threshold currents for any instabilities should exceed the current of this order. Accelerating cavities to be used in a B factory must then be those in which all the higher-order modes are damped out. Furthermore, if a large ring is heavily loaded by the beam, the accelerating mode, itself, will give rise to a coupled-bunch instability for the reason presented in the next section.[1] This report briefly describes a possible solution of this problem.

### II. THE INSTABILITY ARISING FROM THE ACCELERATING MODE

In order to ensure the sufficiently long quantum life time and/or to keep the short bunch length, the beam is accelerated at the phase deviated from the crest. This non-zero synchronous phase gives rise to the reactive component of the electric field in an accelerating cavity. The cavity is usually detuned in order to compensate this reactive component. The amount of the detuning is proportional to the tangent of the synchronous phase  $\phi_s$  and the beam loading  $P_b$  and inversely proportional to the stored energy  $U$  [2]:

$$f_a \left( \frac{f}{f_a} - \frac{f_a}{f} \right) = \frac{P_b}{2\pi U} \tan \phi_s, \quad (1)$$

where  $f_a$  and  $f$  are the resonant frequency of the cavity and the accelerating frequency, respectively. Note that  $f_a$  can be varied by the tuner installed to the cavity, while the accelerating frequency  $f$  is fixed.

If the number of bunches stored in a ring is  $B$ , the beam current has frequency components of  $nBf_r$ , where  $f_r$  is a revolution frequency and  $n$  is an arbitrary integer. Suppose that the beam executes synchrotron oscillation with a frequency of  $f_{\text{synch}}$ . Then, the beam current is modulated, generating the side band in

the frequency spectra:  $nBf_r \pm f_{\text{synch}}$ . If the response of the cavity to the beam current, that is, the impedance of the cavity is higher for the component  $nBf_r + f_{\text{synch}}$ , the cavity induces the electric field in a phase to amplify the synchrotron oscillation. On the other hand, if the impedance is higher for  $nBf_r - f_{\text{synch}}$ , the cavity works as a damper of the oscillation. Since the detuning always lowers the resonant frequency, making the damping component larger than the exciting one, the detuning never gives rise to the growth of the synchrotron oscillation of this mode (Robinson damping[3]).

However, the synchrotron oscillation is possible in other modes. The oscillation phase of one bunch is not necessarily identical to that of the other bunch. The phase of one bunch can be different from that of the following bunch by an amount of

$$\frac{2\pi m}{B}, \quad (2)$$

where  $m$  is an arbitrary integer. This phase difference induces further frequency modulation, resulting in frequency components given by

$$f_{m,n}^{\pm} = nBf_r \pm [mf_r + f_{\text{synch}}]. \quad (3)$$

Again, if the  $f_{m,n}^+$  frequency component of the impedance is larger than the  $f_{m,n}^-$  component, the coupled-bunch synchrotron oscillation with the mode  $m$  grows. For simplicity, consider the case of  $B = h$ , where  $h$  is a harmonic number. When an accelerating cavity is detuned for the beam loading, the mode of  $m = B - 1$  always grows. If the beam loading is relatively light as usual, the amount of detuning is significantly smaller than the revolution frequency, resulting in low growth rate as compared with the radiation damping time.

However, if the beam loading  $P_b$  is very heavy and the synchronous phase is large, the amount of the detuning can exceed the revolution frequency in a large ring. The example of parameters for KEK B-factory[4] is presented in Table 1. In this case, the detuning of the resonant frequency ( $f_a - f = -180$  kHz) is larger than the revolution frequency, causing the extremely fast growth of the instability.

### III. SHINTAKE'S PROPOSAL [5]

In order to suppress the instability it is quite straightforward to decrease the amount of the detuning. It was noted by Shintake[5] that the decrease in the detuning is possible by increasing the stored energy as can be seen from Eq. (1) or from the intuitive insight into the phenomena, since the stored energy

in a cavity represents the inertia of the fields resisting against the disturbance. In the present case the stored energy must be increased by an order of magnitude. It is proposed by Shintake[4] that the storage cavity with extremely high quality factor should be attached to the accelerating cavity. The use of the TE015 mode was suggested for this purpose. The high quality factor of the storage cavity is required in order to increase the stored energy without the increase in the power dissipation. The proposed parameters[5] for the KEK B factory listed in Table 2 reduces the growth rate of the instability to the radiation damping time.

Table 1

Parameters of the LER of the KEK B-Factory

Beam Energy	E	3.5	GeV
Average Beam Current	$i_0$	2.6	A
Revolution Frequency	$f_r$	99.3	kHz
RF Frequency	$f_{RF}$	508.6	MHz
Harmonic Number	h	5120	
Number of Bunches	B	5120	
Momentum Compaction Factor	$\alpha$	$9.3 \times 10^{-4}$	
Longitudinal Damping Time	$\tau_e$	37	ms
Synchronous Phase	$\phi_s$	87.5	deg
Synchrotron Frequency	$f_{synch}$	6.9	kHz
Cavity Voltage	$V_c$	0.6	MV/cavity
Number of Cavities		37	

Table 2

Cavity Parameters (proposed in Ref. [5])

Accelerating Cavity			
Unloaded Q	$Q_a$	$3.8 \times 10^4$	
Impedance	$R_a$	3.2	M $\Omega$
	$R_a/Q_a$	83	$\Omega$
Stored Energy	$U_a$	0.68	Joule
Power Dissipation	$P_a$	57	kW
Beam Loading	$P_b$	67	kW
Storage Cavity			
Unloaded Q	$Q_s$	$2.6 \times 10^5$	
Stored Energy	$U_s$	7.6	Joule
Power Dissipation	$P_s$	93	kW

#### IV. REALIZATION OF SHINTAKE'S PROPOSAL

The most simple system is that which comprises an accelerating cavity and a storage cavity. This system has, however, the following three serious problems. First, the system has two modes namely 0 and  $\pi$  modes. (Strictly speaking this notation of the mode is not exact, since the system is not perfectly periodic. This notation is used only due to its popularity. Also, the definitions of the 0 and  $\pi$  modes become ambiguous for cavities without any beam loading, in which the phases of fields cannot be defined with respect to the beam direction.) If one mode, for example  $\pi$  mode, is used for the acceleration, the other mode, 0 mode in this case, will give rise to the coupled bunch instability. Since the impedance of this parasitic mode is quite high, being similar to that of the accelerating mode, the growth rate of the instability is significantly high.

Second, the stored energy  $U_s$  of the storage cavity should be of an order of magnitude larger than that  $U_a$  of the accelerating cavity as seen from Table 2, although the ratio of the stored energies is not a free, adjustable parameter. The ratio of the stored energy in one cavity to that of the other cavity in the 0 mode is identical to the ratio in the  $\pi$  mode. (In a circuit analog, for example coupled-resonator model,[6,7] the ratio is unity.) Otherwise, the frequency of the one cavity is deviated from that of the other, preventing the energy flow between the two cavities.

Third, the accelerating cavity is heavily loaded by the beam, while the storage cavity not. The energy flow between two cavities may not be sufficiently fast to compensate this unbalanced beam loading.

In order to simultaneously solve the above-mentioned three problems we proposed to locate a coupling cavity in between the accelerating cavity and storage cavity as shown in Fig. 1. The third problem is immediately solved by the famous field stabilizing characteristics of the  $\pi/2$  mode operation. The second problem can also be solved by decreasing the coupling factor  $k_s$  between the storage cavity and the coupling cavity compared with the coupling  $k_a$  between the accelerating and coupling cavities:

$$\frac{U_s}{U_a} = \left( \frac{k_a}{k_s} \right)^2 \quad (4)$$

This method has been successfully used to adjust the stored energy in a bridge coupler of a coupled-cavity proton linac.[8]

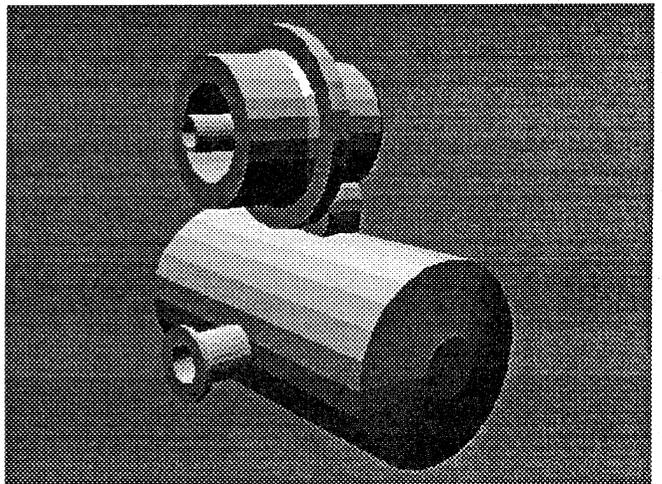


Figure 1. Example of a three-cell cavity for which Shintake's choke-mode cavity[9] is used as an accelerating cavity. A storage cavity is attached from below through a coupling cavity in between.

However, the first problem that is the most serious seems to be worsened by adding the coupling cavity. The number of modes is increased to three, two of which are parasitic modes, appearing to give rise to the instabilities. Here is, fortunately, the strong advantage of the three-cavity system. In the accelerating

$\pi/2$  mode no field exists in the coupling cavity, while strong fields exist in the parasitic 0 and  $\pi$  modes. If one installs a coupler in the coupling cavity and extracts the energy to a matched load, both the 0 and  $\pi$  modes will be efficiently damped, while the accelerating  $\pi/2$  mode will not be affected by the damper.

In addition it should be noted that the resonant frequency of the 0 mode is located at the other side of that of the  $\pi$  mode with respect to the  $\pi/2$  mode as shown in Fig. 2. The frequency separation between the 0 and  $\pi/2$  modes is approximately the same as that between the  $\pi$  and  $\pi/2$  modes. Then, the excitation effect of the instability due to one of the 0 and  $\pi$  modes is cancelled by the damping effect due to the other mode.

The three-cavity system was analyzed[10] by use of the coupled-resonator model[6,7] which has been successfully used to analyze coupled-cavity systems. The result of analysis is summarized in Table 3. The worst possible asymmetry was assumed for passband as shown in Fig. 2. The growths of the instabilities arising from the 0,  $\pi/2$  and  $\pi$  modes are slower than the radiation damping.

### V. CONCLUSION

The three-cavity system comprising the accelerating cavity and storage cavity with the coupling cavity in between is promising for suppressing the longitudinal coupled-bunch instability arising from the accelerating mode. The design of the system is now in progress for the KEK B-factory.

Table 3

Cavity Parameters (proposed in the present paper)			
Coupling between the accelerating and coupling cavities	$k_a$	5	%
Coupling between the accelerating and storage cavities	$k_s$	1.5	%
Asymmetry in the Passband		20	%
Coupling Cavity			
Loaded Q	$Q_c$	22	
Power Dissipation	$P_c$	0.5	kW
Decrease in the total Q value		0.3	%

### VI. REFERENCES

- [1] An Asymmetric B Factory based on PEP Conceptual Design Report (1991).
- [2] P. B. Wilson, SLAC-PUB-2884 (1982).
- [3] K. Robinson, CEAL Report TM-183 (1969).
- [4] S. Kurokawa et al., KEK Report 90-24 (1991).
- [5] T. Shintake, to be published; KEK Preprint 92-191 (1993).
- [6] E. A. Knapp et al., Rev. Sci. Instr. **39**, 979 (1968).
- [7] Y. Yamazaki, Part. Accel. **32**, 39 (1990).
- [8] Y. Morozumi et al., Proc. 1990 Linear Accel. Conf., 153 (1990).
- [9] T. Shintake, Jpn. J. Appl. Phys. Lett. **31**, L1567 (1992).
- [10] Y. Yamazaki et al., to be published; KEK Preprint 93-15.

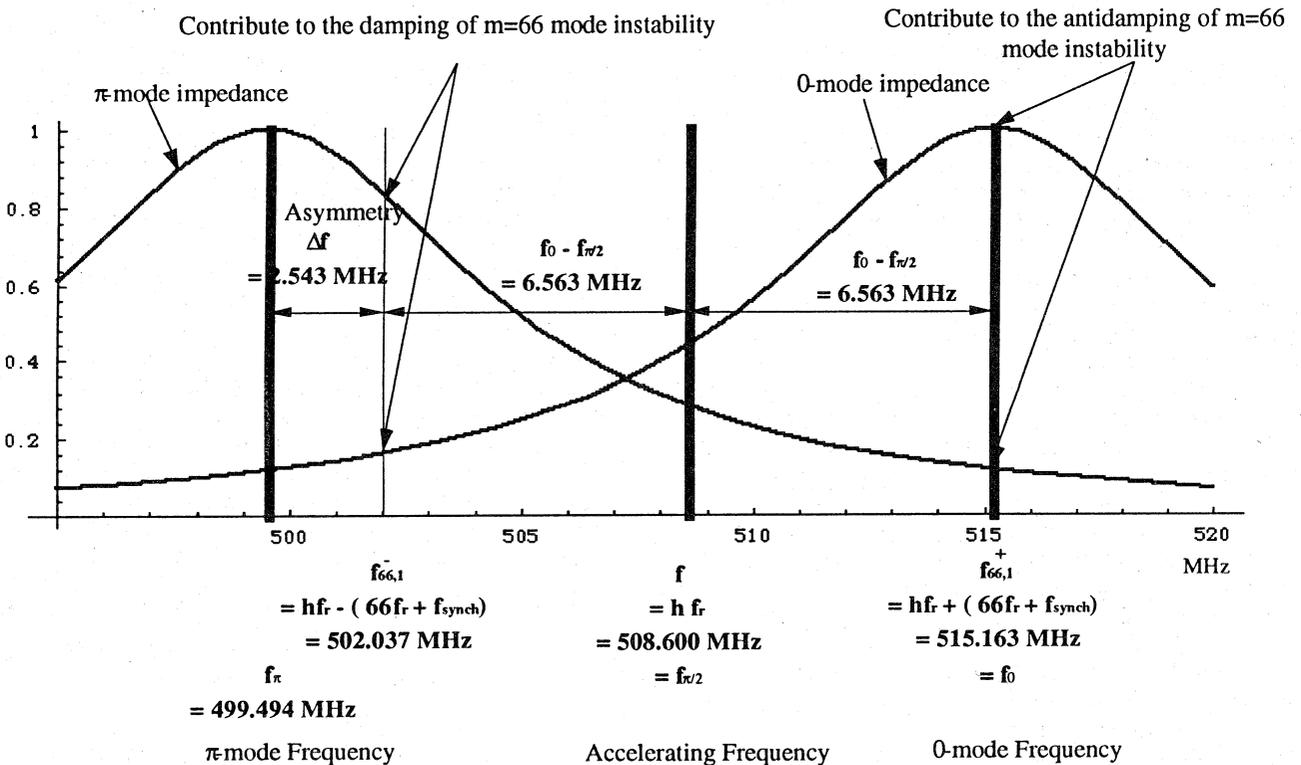


FIGURE 2. The worst example of the frequency spectrum of the impedances of the 0- and  $\pi$ - modes in a three-cavity system. (The impedance of the accelerating  $\pi/2$  mode is not shown, since it is about 500 times higher than those of 0- and  $\pi$ -modes.)