Characteristics of the λ/4 Transmission Line Resonator

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Abstract

Though the spiral cavity is adequate for low frequency operation, mechanical instability becomes serious for such a low frequency as 20 MHz. We have then studied how to shorten the spiral length by using λ/4 transmission line models. Four models with reduced spiral length are presented.

I. Introduction

A linac system for acceleration of short-lived nuclei which are produced at an isotope separator on line (ISOL) is under construction at Institute for Nuclear Study, University of Tokyo. The Linac system is composed of a 25.5 MHz split coaxial RFQ linac and a 51 MHz interdigital-H linac. For matching the longitudinal beam-emittance of the 25.5 MHz RFQ linac to the beam-acceptance of the 51 MHz IH linac, one needs a rebuncher between two linacs. For the operating frequency of the rebuncher, 25.5 MHz is desirable rather than 51 MHz.

Most promising cavity type to keep the cavity size reasonable for such a low operating frequency would be λ/4 transmission line resonator with spiral-shaped inner conductor[1,2]. Even with the spiral cavity, however, one would encounter difficulties arising from long spiral length, 2.94 m for 25.5 MHz.

Thus we have studied how to shorten the length of the inner conductor by using transmission line models. The models differ with each other on the distribution of the capacitance along the line: the capacitance is loaded at the end of the line where the drift tube shall be installed(model A), two transmission lines with different characteristic impedance are connected forming one resonator(model B), the small capacitive elements are distributed along the line(model C), or the hybrid of model A and B (model D). The shortening rate, the Q-value, the shunt impedance have been examined for each model.

In this paper principles of how to shorten the length of transmission line , results of model test , and consideration of the shortening effect are reported.

II. Resonator Models

The spiral type resonant cavity is basically a kind of λ/4 coaxial line resonator. But its outer conductor is divided into two parallel plates and the inner conductor is wound. Considering such a form the transmission line which has two parallel plates and a straight inner conductor between them has been adopted for the test model (Fig.1~4). Though, in the actual spiral cavity, the end of the outer conductor is entirely shorted by the side face of cylinder, it is open in the test model, the effect of which is confirmed to be negligible.

Fig.1
Model A

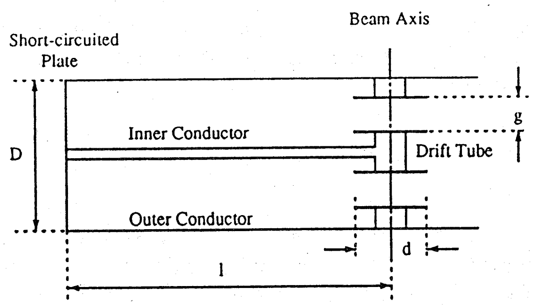


Fig.2
Model B

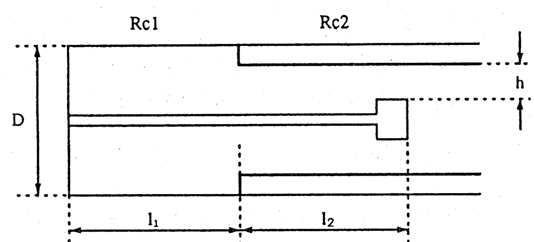


Fig.3
Model C

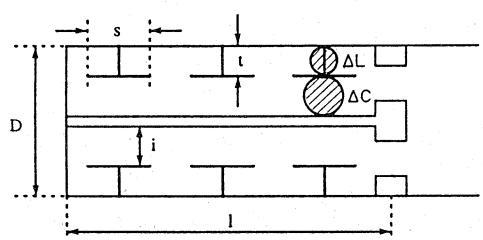
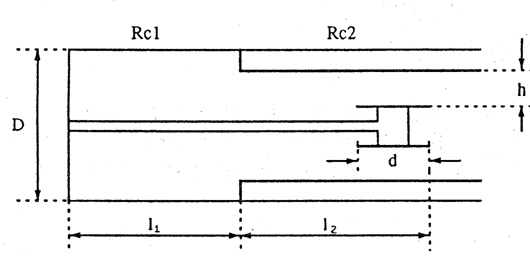


Fig.4
Model D



The model is a half scale of the natural size and is excited by 51 MHz low level rf. All of the parts of the model are made of brass, and are assembled by use of screws. The short-circuiting plate at the left end of transmission line, shown in Fig.1~4, is movable by 2 cm step to shorten the effective transmission line length. The parameter g,h,i in Fig.1,2,3 indicate respective gap spacing. Their minimum value is determined to be 10 mm which is 0.8 times the Kilpatrick limit[3] corresponding to the actual rebuncher cavity.

A schematic view of the models (A), (B) and (C) are shown in Figs.(1), (2) and (3), respectively.

Principles of how to shorten the length of transmission line are as follows.

Model A (Fig.1) : On the normal $\lambda/4$ transmission line, the input impedance Z_1 as seen from arbitrary point A on the line toward the open end O is

$$Z_1 = -jR_c \cot(\beta l_1) = \frac{1}{j\omega C} \quad , \qquad (1)$$

where l_1 is the length AO, Rc is the characteristic impedance of the line, and β is the phase constant of it. Since Z_1 is always negative imaginary, Z_1 is capacitive ($1/j\omega C$). Then a part of the transmission line OA can be replaced by a lumped capacitance C which is determined by the eq.(1). The reduced length of transmission line can be calculated as follows. The input impedance Z_2 as seen from the point A toward the short-circuited end S is

$$Z_2 = jR_c \tan(\beta l_2) \quad , \qquad (2)$$

where l_2 is the length of shortened transmission line. With the resonance condition $Z_1+Z_2=0$,

$$l_2 = \frac{1}{\beta} \tan^{-1}\left(\frac{1}{R_c \omega C}\right) \quad . \qquad (3)$$

Model B (Fig.2) : This model consists of two lines which have different characteristic impedances. The shortened transmission line length of this model is obtainable as follows. The input impedances as seen from the connecting point toward the left and the right are

$$Z_1 = jR_{c1} \tan(\beta l_1) \quad , \qquad (4)$$
$$Z_2 = -jR_{c2} \cot(\beta l_2) \quad , \qquad (5)$$

respectively, where R_{c1} and R_{c2} is characteristic impedance of each line. With the resonance condition Z1+Z2=0, one obtains the equation for the resonant wave length λ,

$$R_{c1} \tan\left(\frac{2\pi}{\lambda} l_1\right) - R_{c2} \cot\left(\frac{2\pi}{\lambda} l_2\right) = 0 \quad . \qquad (6)$$

Solution of eq. (6) under the condition that l_1 plus l_2 is constant shows the length l_1+l_2 is reduced compared with the one for the natural transmission line, in proportion to R_{c1}/R_{c2}. The reduction of the length is largest for $l_1=l_2$.

Model C (Fig.3) : On this model a small plates are distributed along the inner conductor supported from the outer conductor plates. These small plates is expected to work as capacitive elements without decreasing inductance L along the line. Thus the phase constant, shown below, will become large for fixed ω, and the resonant wave length will be longer.

$$\beta = \omega \sqrt{LC} = \frac{2\pi}{\lambda} \qquad (7)$$

Since it is difficult to know modified C and L along the line, we cannot predict the efficiency of line length reduction.

Model D (Fig.4) : This model can be said to be a hybrid of model A and B. The shortening effect is expected to come from both principles of model A and B.

III. Measurements

Fundamental electric constants such as resonant wave length, Q-value, shunt resistance and characteristic impedance have been measured for each model. Characteristic impedance can be known by loading lumped capacitance at the open end of the line and by using the eq.(3). Shunt impedance is simply obtained from the measured shunt resistance multiplied by a factor 2. The conductivity of the models are not perfect and differ among the models.

On model C, the measurement have been done with the ratio of total capacitive elements length to the stem length (C/S) varied from 0.2 to 0.6.

IV. Results of Measurements

The Table 1 shows the variation of shortening ratio (l/l_0), unloaded Q, and shunt impedance (R_{sh}) when parameters are changed on each model. Remarkable features of the results are as follows.

Model A : The line length can be shortened to be 0.45 times the original one (No.5). l/l_0 is nearly proportional to R_{sh}/R_{sh0}, where R_{sh0} is the shunt impedance of the standard (original) model.

Model B : The line length can be shortened to be 0.88 times the original one (No.7). Since R_{c2} is fixed close to the kilpatrick limit, one must increase R_{c1} to get smaller l/l_0.

Model C : The line length can be shortened to be 0.82 times the original one (No.11). When one wish to make shorter, parameter s or t as shown in Fig.3 should be made longer. The later way means that gap spacing between outer conductors D should be expanded.

Model D : This is a hybrid type transmission line of model No.5 and No.7 . The line length can be shortened to be 0.43 times the original one. It is nearly equal to the value in model No.5. The reason of this seems that most part of the shortening effect in model No.7 is already covered by that in model No.5.

Table 1 : l/l_0, Q and R_{sh} for various parameters .

Type	Model No.	Parameters	l/l_0	Q	Rsh(MΩ/m)
Standard Model	1	No Capacitances	1.0	1698	4.37
Model A	2	d=50mm, g=16mm, C=1.63pF	0.95	1625	4.24
	3	d=100mm, g=16mm, C=8.14pF	0.79	1591	3.62
	4	d=150mm, g=16mm, C=19.0pF	0.57	1577	2.68
	5	d=150mm, g=10mm, C=30.4pF	0.45	1519	1.89
Model B	6	Rc1=115Ω, Rc2=95Ω, h=20mm, $l_1=l_2$	0.94	1506	3.39
	7	Rc1=115Ω, Rc2=75Ω, h=10mm, $l_1=l_2$	0.88	1214	2.25
Model C	8	i=20mm, C/S=0.20	0.97	1539	3.84
	9	i=20mm, C/S=0.41	0.95	1449	3.80
	10	i=20mm, C/S=0.61	0.92	1332	3.18
	11	i=10mm, C/S=0.61	0.82	1003	2.47
Model D	12	Rc1=115Ω, Rc2=75Ω, h=10mm, d=150mm, $l_1=l_2$	0.43	664	1.16

Table 2 : Examples of 30% or 50% reduction of the transmission line length.

Type	Examples of Parameters	
	30% Shotening	50% Shortening
Model A	g=10mm, d=80mm,C=8.02pF	g=10mm, d=136mm, C=24.8pF
Model B	Rc1=208Ω, Rc2=75Ω, l1=l2, D=0.5m, h=10mm	Unrealistic
Model C	i=10mm, C/S=0.7	i=10mm, C/S=0.98
Model D	Rc1=115Ω, Rc2=75Ω, $l_1=l_2$, d~80mm	Rc1=115Ω, Rc2=75Ω, $l_1=l_2$, d~136mm

V. Summary

As a summary, examples of 30% or 50% shortenable transmission line models are shown in Table 2.

Concerning model A and B, the measurements agree with the calculations, and the parameters for these example have been determined by using eq.(3),(6), respectively. As for the model C the parameters in the table have been determined by the extrapolation of the results of measurements. As for the model D the parameters has been determined by using the fact that the shortening is almost made by the capacitance attached to drift tube. Through the measurements we have studied how to shorten the transmission line length. It is very important hereafter to examine the shunt impedances exactly for each model. The final model will be determined so as to ensure the mechanical stability and also to keep a reasonable value for the shunt impedance.

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VI. References

[1] A.Schemmp, et al., Nucl. Instrum. Meth. 135 (1976) 409.
[2] P.Z.Peebles, et al., Particle Accelerators 6 (1975) 201.
[3] W.D.Kilpatrick, Rev. Sci. Instrum. 28 (1957) 824.