# Electron Acceleration by Gaussian Laser Beam

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## Abstract

It is shown that the longitudinal electric field of a transverse magnetic mode of a Gaussian laser beam accelerates an electron to an ultrarelativistic energy. The electron is captured and accelerated in a length of the order of the Rayleigh range. The ultimate energy increment of the electron with a single laser beam is given by the product of transverse field intensity and the beam waist, and can be of the order of 100MeV. This fact implies that a multi-stage acceleration enables TeV-order-acceleration in a length of a few kilometers with the art.

## 1 Introduction

The energy density of a laser pulse is usually much greater than that of microwave, and hence many ideas for using powerful lasers to accelerate electrons have been published.[1−3] Since, however, the laser beam is essentially a transverse electromagnetic wave, directions of acceleration in those schemes are transverse and this gives rise to complexity of configurations.

It has been noticed, on the other hand, that the Gaussian beam has a longitudinal field component $E_z(x,y,z)$.[4] Since $E_z$ is connected with the transverse field component $\mathbf{E}_t(x,y,z)$ throught $\nabla\cdot\mathbf{E}=0$, $E_z$ is formally given by

$$E_z = -\int \nabla_t\cdot\mathbf{E}_t dz, \tag{1}$$

where $\nabla_t$ stands for the transverse part of the divergence operator $\nabla$.

Cicchiteilli et. al.[5] ascribed to this field the high energy electron ejection from laser irradiated materials and Scully[6] gave a qualitative discussion on the possibility of using this field for accelerating electrons based on the concept of the linear accelerator. On the other hand, Shimoda[7] gave an elementary methode for calculating field components of a laser beam with finite width and explicitly acquired the longitudinal field. Accordingly we expected an electron acceleration with this longitudinal field and obtained a concrete expression of electron energy gain. We also found that the acceleration mechanism is relevant to the beat wave accelerator[8] rather than to the linear accelerator because of the presence of a phase slippage.

## 2 Gaussian beam

We start with reviewing the methode of Ref.7 and obtaining the required longitudinal field component. Suppose that a TM wave whose frequensy $\omega$ and wave number $k$ is propagating in the $z$-direction in free space with the fields,

$$(B_x, B_y, B_z) = [f(x,y,z), g(x,y,z), 0]\exp(i\omega t - ikz). \tag{2}$$

It is shown that $f$ and $g$ satisfy the following Helmholtz equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ik\frac{\partial u}{\partial z} = 0, \tag{3}$$

under the paraxial approximation $|\partial^2 u/\partial z^2| \ll |2k\partial u/\partial z|$ where $u$ stands for $f$ or $g$. The Gaussian-type solution is written in the form[9],

$$u = \frac{w_0}{w}H_m\left(\sqrt{2}\frac{x}{w}\right)H_n\left(\sqrt{2}\frac{y}{w}\right)\exp(i\Phi_{m,n})\exp\left(-\frac{r^2}{w^2} - \frac{ikr^2}{2R}\right), \tag{4}$$

where $r^2 = x^2+y^2$, $\Phi_{m,n} = (m+n+1)\tan^{-1}(z/\ell_0)$, $F^{-1} = 1/w^2(z) + ik/2R(z)$, $w(z) = w_0\sqrt{1+z^2/\ell_0^2}$, $R(z) = z + \ell_0^2/z$, $\ell_0 = \pi w_0^2/\lambda$, and $H_m$ and $H_n$ are the Hermite polynomials. The $\ell_0$ is the Rayleigh range, $w_0$ is the beam radius at the waist and other notations are standard.

We choose $f = u_{mn}$ and then have $g = -\int(\partial u_{mn}/\partial x)dy$, using the relation $\partial f/\partial x + \partial g/\partial y = 0$ from $\nabla\cdot\mathbf{B}=0$. The best mode for electron acceleration is the one with $m=0$ and $n=1$ or vice versa. We understand this from the following argument. The $E_z$ is easily obtained to be

$$E_z = -i\frac{2Aw_0}{kw^2}\left(1-\frac{r^2}{F}\right)\exp(i\omega t - ikz + i\Phi_{0,1})\exp\left(-\frac{r^2}{F}\right) \tag{5}$$

where we introduced an amplitude $A$ which relates to the maximum amplitude of the transverse electric field given by the relation $A = \sqrt{2e}|E|_{max} = 2.33|E|_{max}$ where $|E|_{max} = \max(\sqrt{|E_x|^2+|E_y|^2})$.

In the vicinity of the beam axis, i.e., $r\approx 0$, the approximations $f\approx g\approx 0$ are valid, then the transverse components of the field are given by $E_x = B_y \approx 0$, $E_y = -B_x \approx 0$. Whereas $E_z$ takes the maximum value. These properties are appropriate for acceleration. We see the mode considered is symmetric around the beam axis and therefore we call it $TM_0$ mode.

## 3 Acceleration mechanism

We suppose that a relativistic test electron is injected from the left-hand side and directed to the right; the direction is the same as the laser beam. Since the transverse field on the axis is zero, we only focus our attention on interaction with the longitudinal field, $\bar{E}_z = Re(E_z)$, at $x=y=0$. With $\zeta \equiv z/\ell_0$, $w^2 = w_0^2(1+\zeta^2)$ and familiar relations, $\sin(2\tan^{-1}\zeta) = 2\zeta/(1+\zeta^2)$, and $\cos(2\tan^{-1}\zeta) = (1-\zeta^2)/(1+\zeta^2)$, we have

$$\bar{E}_z(\psi) = \frac{2A}{kw_0}\left[\frac{2\zeta\cos\psi}{(1+\zeta^2)^2} + \frac{(1-\zeta^2)\sin\psi}{(1+\zeta^2)^2}\right], \tag{6}$$

where $\psi = \omega t - kz + \psi_0$, $\psi_0$ being the phase of a particle to start at $z = -\ell_0$, which is the case through the text. We assume the injected electron has a large $\gamma$, e.g., $10^3$, and then we may approximate $\psi$ by

$$\psi \approx \psi_0 + k\ell_0(\zeta+1)/2\gamma^2 \approx \psi_0, \tag{7}$$

where we have used $t \approx (z+\ell_0)/v_z$ and $(1-\beta_z) \approx 1/2\gamma^2$. Relation (7) implies that for the accelerating electron, $\psi$ is nearly locked at the initial phase $\psi_0$. This fact is already known in the electron linear accelerator in which a longitudinal electromagnetic wave with the velocity of light $c$ is used. We must, however, note that the actual phase is $\psi+\Phi_{01}$ as seen in expression (5) and this point will be discussed later on.

Now assuming that $\gamma$ is large enough, we replace $\psi$ in eq.(6) by $\psi_0$ and then we easily see the motion of electron is governed by the following energy conservation relation

$$mc^2\gamma + U(\zeta) = mc^2\gamma_0 + U(\zeta_0) \equiv K_0, \tag{8}$$

$$U(\zeta) \equiv -\int_{-1}^{\zeta} q\bar{E}_z(\zeta)\ell_0 d\zeta, \tag{9}$$

$$= -qAw_0(\zeta\sin\psi_0 - \cos\psi_0)/(1+\zeta^2). \tag{10}$$

Figure 1 shows a typical profile of $U(\zeta)$.

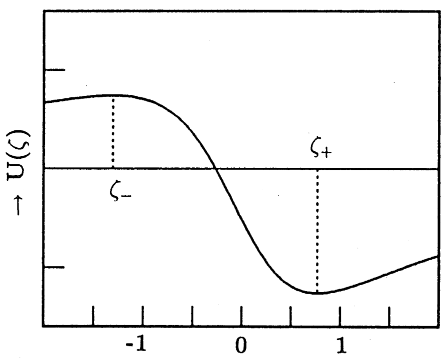

Fig.1

The $U(\zeta)$ has two extremums $U^+ \equiv U(\zeta^+)$ and $U^- \equiv U(\zeta^-)$ which are given by

$$U^{\pm} = (qAw_0/2)(\mp 1 + \cos\psi_0), \tag{11}$$

$$\zeta^{\pm} = (\pm 1 + \cos\psi_0)/\sin\psi_0. \tag{12}$$

The $U^+$ and $U^-$ are the minimum and the maximum, respectively, irrespective of the sign of $\sin\psi_0$. When the initial energy $K_0$ of the electron is greater than $U(\zeta^-)$, the particle can run over the maximum at $\zeta = \zeta^-$ even if $\zeta_0 < \zeta^-$ and the energy gain is expressed as

$$mc^2\Delta\gamma = U(\zeta_0) - U(\zeta). \tag{13}$$

The energy gain between $\zeta = 1$ and $\zeta_0 = -1$, or in the Rayleigh range, is given by

$$mc^2\Delta\gamma = U(-1) - U(1) = qAw_0\sin\psi_0. \tag{14}$$

We understand that the maximum energy gain is achieved when $\sin\psi_0 = 1$. We see, in this case, $\zeta^+ = 1$ and $\zeta^- = -1$, which coincide with the both ends of the Rayleigh range, and there the potential takes its minimum and maximum. For $\sin\psi_0 < 0$, the electron is decelerated as is seen from Eq.(14).

## 4 Longitudinal phase stability

The phase locking between the electron and the longitudinal field is a sufficient condition for a continuous acceleration in the full Rayleigh range. Figure 2 gives critical $\gamma_0$, $\gamma_p$, to the phase-lock approximation for $\sin\psi_0 = 1$ and for various $kw_0$.

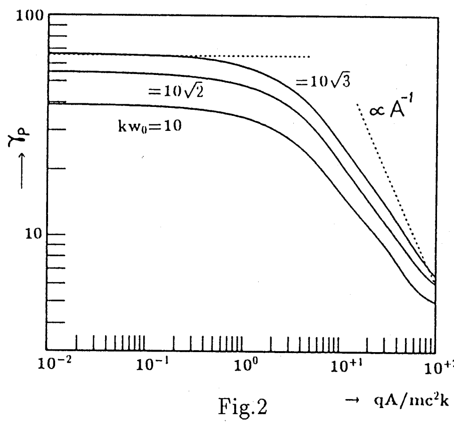

Fig.2

Above the curve the difference between the numerical value $(\Delta\gamma)_{num}$ obtained by using the exact $\bar{E}_z(\psi)$ and the analytical value $(\Delta\gamma)_{an}$ of Eq.(14) is less than 1%. The $\gamma_p$ levels off for small amplitude $A$ as seen in the figure. This is because the test electron just passes through the Rayleigh range with little interaction with the field having such a small amplitude. We can estimate $\gamma_p$ in this limit analytically and have $\gamma_p = kw_0(\pi-2)^{1/2}/(2\sqrt{2\varepsilon})$ which agrees with the numerical one, where $\varepsilon = |1-(\Delta\gamma)_{an}/(\Delta\gamma)_{num}|$. Dotted lines in the figure show analytical results.

We consider the matching between the particle and the wave. The phase velocity $v_p$ is obtained from relation $\omega t - kz + \Phi_{01}$ =constant as

$$v_p = \frac{c}{1-2\ell_0/k(z^2+\ell_0^2)} \approx c\left[1+\frac{2}{k\ell_0(1+\zeta^2)}\right]. \tag{15}$$

Obviously $v_p$ is greater than $c$ and then the accelerating particle sooner or later falls down into the deceleration phase. To be precise, we notice that $2\tan^{-1}(1) = \pi/2$ and $2\tan^{-1}(-1) = -\pi/2$ and then from Eq.(7) the phase $\omega t - kz + \Phi_{01} \approx \Phi_{01}$ differs by $\pi$ between $\zeta_0 = -1$ and $\zeta = 1$ or between the initial and the final phases. This means that the particle is accelerated in the half wave length. In this sense the acceleration scheme presented here is essentially different from the usual electron linear accelerator in which electrons can be accelerated indefinitely.

## 5 Tansverse stability

So far we have assumed that the test electron runs along the beam axis on the center. Consequently the particle is not affected by the transverse field because $B_x$ and $B_y$ are zero on the axis and so $E_x$ and $E_y$. When the particle is injected off the axis, however, it may be deflected by the transverse fields. We then examine the stability of particle orbit. We deal with the motion on the $x-z$ plane, since the laser beam of $TM_0$ mode is symmetric around the axis. We inject the test particle at the point $(w_0/\sqrt{2}, 0, -\ell_0)$, bearing in mind that $|E_x|$ takes its maximum at $(w_0/\sqrt{2}, 0, 0)$. The $x$ component of the force, $F_x$, affecting the electron is

$$F_x = qE_x - (q/c)B_y v_z \approx qE_x/(2\gamma^2). \tag{16}$$

A similar force $qE_y/(2\gamma^2)$ acts on the electron on the $y-z$ plane. We examine the orbit for $\sin\psi_0 = 1$ for which the particle is continuously accelerated. If the electron is not deflected immediately after the injection, there occurs no deflection afterwards since $\gamma > \gamma_0$ and since $|F_x|$ rapidly decreases. Suppose that we reduce $\gamma_0$ and that for some $\gamma_0$, say $\gamma_d$, the electron starts to be deflected near $\zeta_0 = -1$. We call $\gamma_d$ the marginal $\gamma_0$ that divides the motion into the stable and the unstable ones.

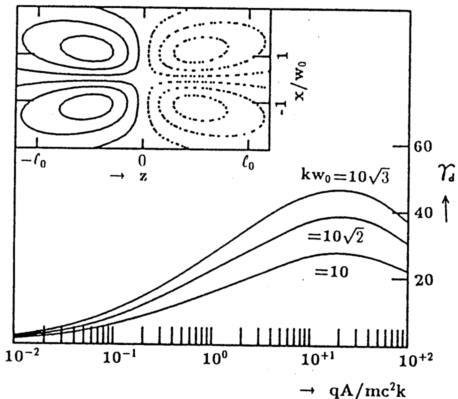


Fig.3

The $\gamma_d$ is given in Fig.3 together with the filed $E_x$ in the inlet. Each curve has its maximum. This is caused by a property of $F_x$ in Eq.(16). That is, for a small $A$, $F_x$ is proportional to $E_x$ or $A$, while for a large $A$, $F_x$ is inversely proportional to $A$. Now we understand that if we use a laser with a $kw_0$ an electron whose $\gamma_0$ is above both curves labeled with the same $kw_0$ in Fig.2 and in Fig.3 is stably accelerated.

## 6 Discussion

We show here some examples of electron acceleration. Suppose that $\gamma_0 = 10^3$ (510MeV of the electron) which safely clears the above criterions irrespective of $kw_0$ and that $\lambda = 10^{-3}$cm ($CO_2$ laser). The maximum energy gain $G_{max} \equiv mc^2(\Delta\gamma)_{max}$ of the electron is given by Eq.(14) with $\sin\psi_0 = 1$. The $A$ relates to $|E|_{max}$ as indicated previously and then we easily obtain $G_{max}$ in terms of $|E|_{max}$. We here express $G_{max}$ in term of the power $P$ of the beam as follows:

$$G_{max} = 0.64 \times 10^2 \left[P(\mathrm{W/cm^2})w_0^2(\mathrm{cm})\right]^{1/2} \mathrm{eV}. \tag{17}$$

We now note that $P \times \pi w_0^2$ is constant provided the total power is fixed. This implies that $G_{max}$ is constant in this constraint no matter how we control $w_0$, while the acceleration length $R_a \equiv |\zeta^- - \zeta^+| = 2\ell_0 = kw_0^2$ is reduced if we have a stronger focus. We now choose $P = 10^{14}$ W/cm$^2$, a realistic value in view of the present state of the art in laser technology, and $w_0 = 0.1$ cm. Then we have $G_{max} = 6.4 \times 10^7$eV and $R_a = 63$cm. If we use the those values, we will have 1TeV electrons in a 10 km long device that is made of $16 \times 10^3$ units of the acceleration element with 63cm long. With the same power and with $w_0 = 0.033$cm, the $R_a$ reduces to 6.3cm and the total length of the 1TeV accelerator will be reduced to 1km though the focused electric field will increase to $10^9$V/cm, still within the present state of the art of laser technology.

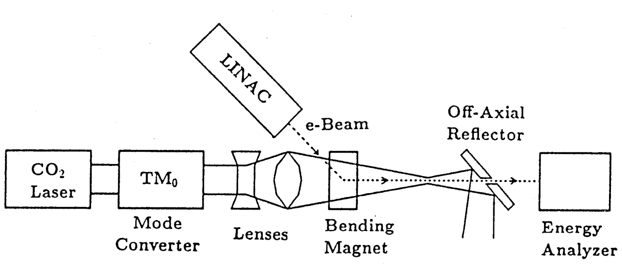


Fig.4

A schematic diagram of an experiment to verify the acceleration mechanism presented here is shown in Fig.4.

## 7 Conclusion

We have proposed a new scheme of electron acceleration by using the longitudinal field accompanying a laser of finite width. We first derive analytically Hermite-Gaussian laser beams by using the paraxial approximation. The field components properly include a longitudinal electric field whose intensity is maximum at the beam axis while the transverse components become zero there. A test electron is phase-locked and stably accelerated when $\gamma_0 > \gamma_p$ which corresponds to the minimum injection energy and also when $\gamma_0 > \gamma_d$ above which the deflection by $E_x$ and $E_y$ is negligible. It is shown that the maximum energy increment is independent of the injection energy of a particle and is given by $qAw_0$. A numerical demonstration for applying this scheme to an accelerator is made and we find that it is plausible to have an 1TeV electron accelerator with a few kilometers length by the use of lasers of the present state of the art.

## References


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