SNAKE AND ROTATOR DESIGN FOR POLARIZED PROTON BEAMS IN THE PS COLLIDER

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Abstract

A compact snake and rotator scheme for polarized proton beams, which can provide variable spin-rotation axis with small orbit displacement, is proposed. This aims to maintain polarization and to obtain both longitudinal and transverse polarization at the collision point in the heavy ion collider (PS-Collider), which has been recently proposed as one of the future projects of the KEK PS.

Introduction

A "polarized collider" has been proposed as a option of the so-called PS Collider (PSC). In order to create high nuclear density matter ($\rho \sim 10\ \rho_0$), the PSC aims to accelerate and collide various ions from proton to gold at the energy of 5 ~ 7 GeV/u with a luminosity of 10^{25} cm^{-2} s^{-1}.[1,2,3,4] The PSC can be also used to collide polarized protons as a "polarized collider", which can provide polarized protons of much higher energy than presently available energies. The proton kinetic energy of the PSC is up to 19 GeV, corresponding to the beam energy of ~ 845 GeV in the fixed target experiment. One of the most interesting physics with the "polarized PSC" is to measure gluon polarization, which may solve the "spin crisis".[5,6] Many other interesting physics can be done with this machine.[5,6]

The PSC is a superconducting accelerator and collider in order to make it possible to be located in the present East Experimental Hall of the KEK 12 GeV proton synchrotron (PS). The existing PS will be used as an injector of the PSC. The beams from the PS will be injected into two identical rings of the PSC at the energy of 1.9 GeV (0.5 GeV) for proton (gold), accelerated up to the top energy and debunched for colliding experiments with a horizontal crossing angle of 1.8 x 2 degree. The reason why the scheme of coasting beam collision is adopted, is to reduce the beam blow up due to intrabeam scattering in the case of heavy ions like gold.[7]

If we choose the same lattice and the same collision scheme as the gold collision, the achievable luminosity for the polarized protons will be 10^{27}-10^{28} cm^{-2} s^{-1} and the 40-60 % polarization is expected. These are limited by the injected beam intensity and polarization. The design principles are:

1. Maintain polarization against many depolarizing resonances.
2. Realize longitudinal and transverse polarization at the collision point.
3. Use the lattice of heavy ions collision.

In order to fulfill the above principles, we adopt a Siberian snake scheme with compact snakes and rotators proposed here.

Polarization with snakes and rotators in the PS Collider

There are 65 intrinsic resonances and 33 imperfection resonances in the PSC. Such many resonances are due to the superperiodicity of one, which comes from coasting beam collision with a crossing angle. The strength of intrinsic resonances range from 0.0005 to 0.06, using the vertical emittance of 8 π mm mrad at injection and the vertical betatron tune of 7.3 (without high transition energy operation). The strength of imperfection resonances are between 0.0001 and 0.02 with the vertical closed orbit distortion of 1 mm in rms. It is formidable to correct such a great number of resonances by conventional methods as fast tune jump and harmonic control of closed orbit distortion. Moreover we need longitudinal and transverse polarization at the collision point. Therefore, we adopt a "Siberian snake" scheme.[8,9]

This method is to introduce the magnet sequences, called "snake", into certain places in the ring. Each snake keeps the orbit unchanged and rotates the spin by 180 degree about the axis in the horizontal plane. Then they can cancel all depolarizing resonances, if certain conditions are satisfied. This process makes the spin tune (spin precession frequency per one revolution) 1/2 and keep away from all depolarizing resonances. This snake can act as a rotator, i.e. provide longitudinal and transverse polarization at the collision point, with appropriate arrangement.

In order to obtain vertical polarization in the arcs, the number of snakes should be even number.[10] Provided even number (N) of snakes, the following equation should be fulfilled to realize the Siberian snake scheme:[10]

$$\sum_{j=1}^{N} (-1)^j \gamma G\,(\theta_{j+1} - \theta_j) = \sum_{j=1}^{N} (-1)^j\, \psi_j = 0, \qquad (1)$$

$$\nu_{spin} = \frac{\sum_{j=1}^{N} (-1)^j\, \alpha_j}{180} = 0.5\ , \qquad (2)$$

where γ is the Lorentz factor, G is the anomalous part of the proton magnetic moment, the j-th snake has the precession axis of α_j degree measured from the radial axis, the spin precession angle between the j-th and the (j+1)-th snakes is ψ_j degree, θ_j is the azimuthal angle of the j-th snake and ν_{spin} is the spin tune. Eq.(1) guarantees energy independence of the spin tune (ν_{spin}) and Eq.(2) leads the spin tune 1/2.

We can realize longitudinal and transverse polarization at the collision point with 2 snakes, provided $\theta_1 = \theta_2 = \pi$ and $\alpha_1 - \alpha_2 = 90$ degree according to Eqs (1) and (2). The schematic drawing is shown in Fig.1. In the upper of the ring, the snake is split into two parts. Each part rotates the spin by 90 degree about the axis in the horizontal plane in order to provide longitudinal polarization. The bending magnets which deflect the beam by 1.8 degree for collision, rotate the spin by γG x 1.8 degree around the vertical axis. Because there is a relation between the orbit and the spin rotation as follows,

$$\psi = \gamma G\, \theta, \qquad (3)$$

which is reduced from Thomas-BMT equation,[11] where ψ is spin precession angle, θ orbit deflection angle. The precession axis of the split snake should be located in order to compensate this undesirable precessions as indicated in Fig.1. The angle of the snake axis measured from the radial axis runs from 9.9 to 68.6 degree as the energy changes from 1.9 to 19 GeV for the upper snake. The snake shown in the lower side should have an angle from –80.1 to –21.4 degree due to Eq.(2).

In order to realize Siberian snake scheme, the spin tune spread ($\Delta\nu_s = \nu_s - 1/2$) should be small enough to keep the spin tune away from the resonances. Because with perturbations, which produces depolarizing resonances, the spin tune deviates from 1/2. The spin tune spread due to these perturbations is[12]

$$\Delta\nu_s = \frac{1}{\pi} \sin^{-1} \left[\frac{|\varepsilon|^2}{\delta^2 + |\varepsilon|^2} \sin^2 \frac{\pi (\delta^2 + |\varepsilon|^2)^{\frac{1}{2}}}{N} \right], \qquad (4)$$

where

$$\delta = \kappa - \gamma G,$$

κ is a resonance frequency and ε is a resonance strength. In our case N = 2 and ε is less than 0.06. The perturbed spin tune spread $\Delta\nu_s$ is less than 0.01 and does not raise any problem.

Another problem is the deviation of the spin from the equilibrium axis during resonance passage, although final polarization is maintained. The envelope equation is given as[12]

$$\langle s \rangle = 1 - 8\, a^2 b^2, \qquad (5)$$

where

$$a = (1 - b^2)^{1/2},$$

$$b = \frac{|\varepsilon|}{(\delta^2 + |\varepsilon|^2)^{\frac{1}{2}}} \sin \frac{\pi (\delta^2 + |\varepsilon|^2)^{\frac{1}{2}}}{N}$$

The deviation becomes maximum on the resonance. Substituting the parameters of the PSC, we get <s> ~0.93 for ε ~0.06, which is the maximum case. From the point of view of physics experiment, polarization is almost fully kept except a little degradation at those stronger resonances.

Although two snake scheme seems to give a good solution for our purpose, it is very difficult in practice to realize those scheme in lower energy region of the PSC (1.9~10 GeV in kinetic energy), because of difficulty to get type II snake, i.e. the overall spin rotation is 180 degree about the radial axis, or near type II snake with small orbit displacement.

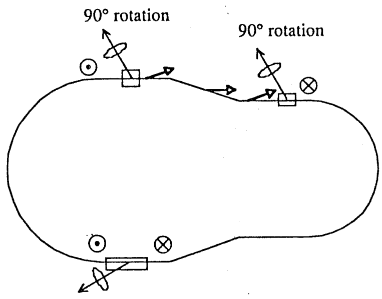

Fig.1 The schematic drawing of the polarized PCS.
-θ> indicates the snake axis.

Here Siberian snake by two steps is considered. The first step is from 1.9 to 10 GeV, where one snake is energized around type I snake, i.e. the overall spin rotation is 180 degree about the longitudinal axis. When the energy reaches ~10 GeV, the second snake is adiabatically energized.

There are two practical ways to realize a Siberian snake scheme; one is to use a solenoid magnet and the other is to use transverse fields. The former has only a longitudinal snake axis and requires the field strength proportional to the beam energy. Therefore the latter is discussed in the next section.

Proposals of snake and rotator scheme

Many snake and rotator schemes with transverse fields have been proposed by many authors so far. These are consisted of some transverse fields, placed in irreversible way for spin and reversible for orbit, using the relation between the orbit and the spin rotation (Eq.(3)).

Derbenev and Kondratenko (D&K) proposed three types of snakes without orbit correction bends.[9] The first is a type I snake with the following magnet sequence:

$$(-B_V, 2B_H, 2B_V, -2B_H, -B_V),$$

where B_H and B_V are horizontal and vertical bending magnets respectively. The second is a type II snake with the following magnet sequence:

$$(B_{30}, B_{-90}, B_{180-30}),$$

where the suffix means the tilt angle of the spin rotation axis about the longitudinal axis. The last one is a helical wiggler, which has the magnetic field as:

$$B_x = B_{0x} \cos (ks),\ B_z = B_{0z} \sin (ks),\ B_s = 0,$$

where B_x, B_z and B_s are horizontal, vertical and longitudinal component of the magnetic field respectively and s is a position along the longitudinal axis.

The first type is expanded by Steffen as families with variable snake axis, the angle of which ranges −180~+180 degree.[13] They have two kind of orbit correction bending magnet groups. One is a "standard-type" and the other is a "novel-type". The standard-type has a magnet sequence as:

$$(-B_H, B_H)+(-B_V, 2B_H, 2B_V, -2B_H, -B_V)+(-B_H, B_H),$$

and novel-type has a magnet sequence as:

$$-B_H+(-B_V, 2B_H, 2B_V, -2B_H, -B_V)+ B_H.$$

Multi cell snake using this type are considered from the point of view of reducing orbit displacement by Mane.[14] Geometrical economy; spacing optimization, split configuration and so on, are pursued by S.Y.Lee.[15]

Snake using helical wiggler is refined by Courant.[16] The economical version, i.e. descretized helix, is proposed by Wienands.[17] The magnet sequence of one cell is:

$$(B_{180+45}, B_{180-45}, B_{45}, B_{-45}),$$

for the Wienands snake.

Both snakes have horizontal correction bends at both ends. The projection of the orbit to x-z plane looks like rectangle (by thin approximation) in Wienands snake. Triangle version with tilted three bends, corresponding to the second snake of D&K, polygon version and so on are also possible as approximations of a helical wiggler. We can also consider the first snake of D&K and Steffen snake as one of approximations of a helical wiggler.

Summarizing these snakes, using sin-like (cos-like) orbit trajectory in x- (z-) plane, we obtain any snake precession angle and any snake precession axis. The angle of the snake axis can be changed by varying the ratio of the horizontal and vertical field strengths. This means the ratio of ψ_x, ψ_z for D&K and Steffen snake, where ψ_x and ψ_x are the spin precession angle about the field of horizontal and vertical bends respectively, the tilted angle of the bends for Wienands type[18] and the ratio of the horizontal and vertical field amplitude for (elliptical) helical wiggler. The properties are tabulated in Table.1 for the case of D&K and Steffen snake. Here n is a cell number, ψ_s is the spin precession angle about the longitudinal field, $\phi_s^{(stand)}$ and $\phi_s^{(novel)}$ are the angles of snake axis measured from radial axis, D_x and D_z are orbit displacements in the horizontal and vertical plane, L_{body} and L_{corr} are the length of the body of the snake per cell and the length of the correction bending magnets, and l_x and l_z are the lengths of horizontal, vertical bends and l_g is the length of free space between magnets. The other types of snake has similar properties but some need numerical calculations.

Table. 1
Properties of n-cell Steffen snakes

properties	Steffen standard-type	Steffen novel-type
ψ_x, ψ_z	$\sin \psi_x \sin \psi_z = \pm \sin(\pi/4n)$	$\sin \psi_x \sin \psi_z = \pm \sin(\pi/4n)$
ϕ_S	$\cos \phi_S^{(stand)} = -\frac{\sin \psi_x \cos \psi_s}{\sqrt{\sin \psi_x^2 \cos \psi_s^2 + \cos \psi_x^2}}$	$\phi_S^{(novel)} = \phi_S^{(stand)} - \psi_x$
D_x	$2\ (l_x + l_z + l_g)\ \psi_x / \gamma G$	$(l_x + l_z + 2\ l_g)\ \psi_x / \gamma G$
D_z	$(2\ l_x + l_z + 2\ l_g)\ \psi_z / \gamma G$	$(5 l_x/2 + l_z + 3\ l_g)\ \psi_z / \gamma G$
L_{body}	$4 l_x + 4\ l_z + 4\ l_g$	$5\ l_x + 4\ l_z + 6\ l_g$
L_{corr}	$2 (3\ l_x + l_z + l_g)$ (for $\psi_x \le \psi_z$)	$2\ l_x$
	$2 \sqrt{l_g^2 + 8 (l_x + l_z + l_g)\ l_x}$ (for $\psi_x > \psi_z$)	

If the horizontal and vertical field strengths are the same, the orbit displacement is minimum and decreases as the cell number increases. For the helical wiggler, orbit displacement is obtained as

$$r = \frac{1}{k^2 \rho} = \frac{4n+1}{4n^2} \frac{(B\rho)}{(\gamma G)^2 B}, \tag{6}$$

using the relation between Bl and cell number (n)

$$BL = \frac{(B\rho)}{\gamma G} \pi \sqrt{4n+1}, \tag{7}$$

and

$$kL = 2\pi n, \tag{8}$$

where k is the pitch of helical wiggler.

Using these properties, we get the snakes with variable snake axis from 21.4 ~ 68.6 degree. But maximum field is limited by 4 ~7 Tesla, even if super conducting magnets is used. With limited magnetic field, multi snake is preferable in order to realize small orbit excursion. The vertical bends, however, need much stronger field than minimum field to obtain variable snake axis. This vertical field becomes stronger as cell number increases in case of the Steffen novel-type snake and it does not decrease as cell number increases for the Steffen standard-type snake. Assuming B=6.5 T and l_g=0.24 m, the orbit displacements for the Steffen standard-type and the Steffen novel-type snake is obtained as shown in Fig.2. When magnets are energized to realize proper polarization direction at the collision point, as example -21.4 degree at 19GeV, the maximum orbit displacement changes as the curve marked by circles in Fig.2. When magnets are energized to realize minimum orbit displacement without any care about polarization direction, the orbit displacement changes as the curve marked by crosses in Fig.2. The dashed line in Fig.2 indicates the available displacement of 40 mm, according to the present technology of superconducting magnet. It seems difficult to realize the requirement for snake discussed above. We will solve this problem in the next section.

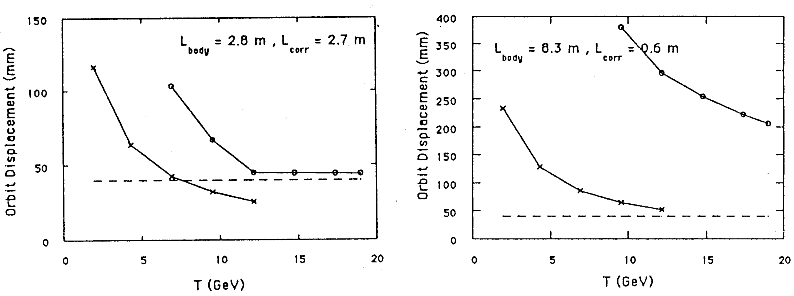

Fig.2 The orbit displacements of Steffen standard-type snake (left) and Steffen novel-type snake (right).

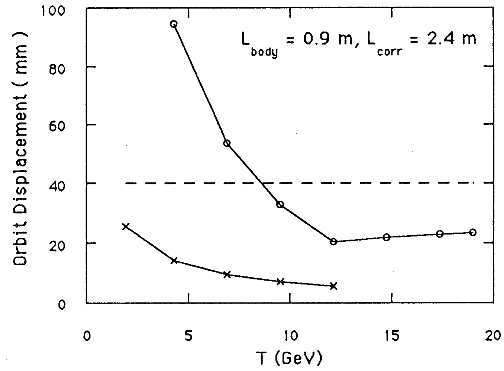

Fig.3 The orbit displacement of new snake.

New snake and rotator scheme

Adding horizontal correction bends with opposite sign and equal magnitude ($\pm\phi_H$) to the snake with precession angle of ϕ and axis of ϕ_s at the both ends, the overall rotation is obtained using spinor matrix as

$$e^{-i\frac{\phi'}{2}(\cos\phi_s'\sigma_x+\sin\phi_s'\sigma_s)} = e^{i\frac{\phi_H}{2}\sigma_z}\, e^{-i\frac{\phi}{2}(\cos\phi_s\sigma_x+\sin\phi_s\sigma_s)}\, e^{-i\frac{\phi_H}{2}\sigma_z} . \qquad (8)$$

We find new snake has a precession angle of $\phi' = \phi$ and an axis of $\phi_s' = \phi_s - \phi_H$ as a whole. Therefore these horizontal correction bends are usable not only for orbit correction but also for variable snake axis without changing a body of the snake. We can choose the most compact snake configuration as a body of the snake. Making use of this property, we obtain rather compact snakes as a whole.

For the PSC multi cell snake is preferable as discussed above in order to reduce orbit displacement. The four-cell snake is investigated here. This snake has the following magnet sequence:

$$-B_{H1} + (-B_{H2}, \text{wiggler}, B_{H2}) + (-B_{H2}, \text{wiggler}, B_{H2}) + (-B_{H2}, \text{wiggler}, B_{H2}) + (-B_{H2}, \text{wiggler}, B_{H2}) + B_{H1},$$

where B_{H1} and B_{H2} are horizontal correction bending magnets and B_{H1}'s are not necessary between the snake cells. Each snake cell is split into the spaces between lattice quadrupole magnets. Providing an spin rotation angle (45 degree per cell in this case) and possible magnetic field, the length and pitch of the helical wiggler are obtained by Eq.(7). Using the requirements for the snake axis direction and the condition of orbit restoration outside the snake, the position and the kick angle of correction bends can be obtained, resulting rather lengthy equations. Assuming B=6.5 T and the space between magnets is 0.24 m, we obtain the parameters of this snake according to the procedure above. The orbit displacement in the snake is shown in Fig.3. When magnets are energized to realize proper polarization direction at the collision point, the maximum orbit displacement changes as the curve marked by circles in Fig.3. When magnets are energized to realize minimum orbit displacement without any care about polarization direction, the orbit displacement changes as the curve marked by crosses in Fig.3. The dashed line in Fig.3 indicates the available displacement of 40 mm as in Fig.2. The one cell length of the wiggler is 0.9 m and the length of the horizontal correction bending magnets is 2.4 m including the free space between B_{H1} and B_{H2}. The length is much shorter than the Steffen's snakes. In order to install this snake, split into each lattice cell, the span between two adjacent quadrupole magnets should be greater than 4.1 m. Only a little modification is needed in the lattice cell structure because the span is 3.7 - 4.3 m in the present lattice. One more lattice cell is necessary for each quadrant of the ring to keep space for injection and acceleration. Applying adiabatic transition from one snake to two snakes around the energy of 10 GeV, this snake realizes small orbit displacement enough to fulfill the magnet aperture limit.

We achieve a practicable snake, which fulfill the requirements of physics experiments, although even using this scheme realizing type II or near type II snake with small orbit excursion is difficult in lower energies of the PSC.

Conclusions

The compact snake with variable snake axis is proposed, making use of horizontal correction bends. This scheme can realize variable snake axis with small orbit displacement. Using this scheme, the preliminary design of snake was performed for the PSC. One snake is energized in the energies of 1.9 - 10 GeV and around 10 GeV the other snake is energized adiabatically. The polarization axis lies on the horizontal plane at the collision points during one snake on. After two snakes on, both longitudinal and transverse polarization are obtained at the collision points. The orbit displacement is less than 30 mm in the snake. Depolarization will be less than 7 % and this snake and rotator scheme in the PSC seems to arise no serious problem.

Further investigation is essential especially of lattice optimization, superconducting magnets design, detailed analysis of orbit and polarization stability.

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