ANALYSIS OF THE SECONDARY EFFECT OF A CORRECTOR MAGNET

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Abstract

A corrector magnet has a secondary effect of kicking the beam to a different direction from the designed direction. It has been difficult to analyze this effect due to its small signal-to-noise ratio.

A new method of analyzing the secondary effect of a corrector magnet was developed. By this method the kick angle of a secondary direction is estimated by least squares and its significance is examined by t-test combined with the outlier detection.

This method was applied to the correctors in SPEAR storage ring. The correctors estimated to have significant secondary effects were found to be rotationally misaligned.

1 Introduction

Corrector magnets are used for correcting the closed orbit of a synchrotron. Although a corrector is designed for kicking the beam to a direction, say horizontally, it has a secondary effect of kicking the beam to the other direction, vertically. The orbit correction should take into account this secondary effect, but it has been difficult to analyze this effect. The difficulty lies in the fact that the secondary kick angle is several orders of magnitudes smaller than the primary kick angle and mixed with the noise.

The author has developed a method of analyzing the secondary effect. This method is based on statistical estimation and testing. This method was applied to the actual correctors of SPEAR storage ring in Stanford. The results were examined by comparing the calculated kick angel with the rotational misalignment of the correctors.

In this paper the analysis method, the results of analysing the SPEAR correctors, and the comparison with the actual misalignment will be presented.

2 Method

In this chapter I will take a horizontal corrector whose primary kick is in the horizontal direction. Suppose there are m beam position monitors (BPM) in the ring. The changes of the vertical beam positions at BPM's $\Delta y_1, \Delta y_2, \ldots \Delta y_m$ are measured when the horizontal corrector is tweaked.

2.1 Estimation of Kick Angle

In this section the unknown kick angle θ in the vertical direction is estimated. The changes of beam positions are related to the kick angle as follows[1]:

$$\theta\vec{a} = \vec{\eta}, \tag{1}$$

where $\vec{\eta}$ is a vector calculated from the beam positions $\Delta y_i\,(i = 1, \ldots, m)$ and the beta functions at the BPM $\beta_i\,(i = 1, \ldots, m)$

$$\vec{\eta} \equiv \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{pmatrix}, \tag{2}$$

$$\eta_i \equiv \frac{\Delta y_i}{\sqrt{\beta_i}}\ (i = 1, \ldots, m). \tag{3}$$

$\vec{a}$ is a vector whose componenents $a_i\,(i = 1, \ldots, m)$ are defined by

$$a_i \equiv \frac{\sqrt{\beta_c}}{\sin \pi\nu} \cos\left(\pi\nu - |\psi_i - \psi_c|\right), \tag{4}$$

where β_c and ψ_c are the beta function and betatron phase, respectively, at the position of the corrector.

The equation (1) should be modified so as to introduce the random errors ε occurring in the measurements

$$\vec{\eta} = \theta\vec{a} + \vec{\varepsilon}. \tag{5}$$

An estimator of the kick angle $\hat{\theta}$ may be calculated by minimizing the following error function:

$$S(\theta) \equiv ||\vec{\eta} - \theta\vec{a}||^2, \tag{6}$$

where $||\vec{b}||$ denotes the absolute value of a vector $\vec{b}$. This solution is known as the linear least squares estimator[2] which is calculated from

$$\left.\frac{\partial S}{\partial \theta}\right|_{\theta=\hat{\theta}} = 0, \tag{7}$$

and the following formula may be derived

$$\hat{\theta} = \frac{(\vec{a}, \vec{\eta})}{||\vec{a}||^2}, \tag{8}$$

where $(\vec{b}, \vec{c})$ denotes the inner product of two vectors.

2.2 Significance Test

The following assumptions were set for the errors ε in the equation(5):

1. The errors $\varepsilon_i (i = 1, \ldots, m)$ are independent of each other.

2. The errors are generated from an identical normal distribution of the average 0 and variance σ^2.

It is known that under these assumptions the following stochastic variable follows the t-distribution[2]:

$$t \equiv \frac{\hat{\theta}}{\sqrt{\frac{S_0}{m-1}}} \tag{9}$$

where S_0 is the residual sum of squares defined by the following formula:

$$\begin{aligned} S_0 &= \|\vec{\eta} - \hat{\theta}\vec{a}\|^2 \\ &= \|\vec{\eta}\|^2 - \frac{(\vec{a}, \vec{\eta})^2}{\|\vec{a}\|^2}. \end{aligned} \tag{10}$$

The test procedure is as follows:

- If t in the equation (9) is larger than the critical value t_c, $\hat{\theta}$ is statistically significant, i.e. the corrector has a secondary effect.
- Otherwise, the corrector's secondary effect is negligible.

The critical value t_c is calculated as a function of the degree of freedom of the data φ and the significance level α which is given by the user

$$t_c = t_c(\varphi, \alpha) \tag{11}$$

$$\varphi = m - 1. \tag{12}$$

2.3 Exclusion of Outliers

When the error is not regular, the signal may not be picked up even by statistical method described in the previous section. A method of excluding the "outliers", i.e. abnormal data is necessary.

Several methods were tried to exclude the outliers. Those are:

1. Exclude the most different data from the regression

$$\max_i |\eta_i - \hat{\theta} a_i|. \tag{13}$$

2. Exclude such data that the residual set of data give the maximum t value.
3. Exclude such data that the residual set of data give the minimum residual sum of squares (S_0).

It was observed that method 1 does not effectively increase the t value, and method 2 does not always exclude the obvious outliers. Therefore, method 3 was selected.

3 Application

The method described in chaper 2 was applied to all the corrector magnets installed in SPEAR storage ring in Stanford.

SPEAR has 26 horizontal correctors, 27 vertical correctors, and 25 BPMs. The horizontal correctors are 22 trim coils of the bending magnets named $*BB*T$ and the four independent corrector magnets named $HCORR*$ installed in the straight sections. The vertical correctors are 23 trim coils of the quadrupole magnets named $*QF*T$ or $*QD*T$, and the four independent corrector magnets named $VCORR*$ installed in the straight sections.

The outliers were excluded until the t value become larger than the critical value of five percent significant level $t_c(\varphi, 0.05)$. The correctors with less than ten outliers were judged to have significant secondary effect.

Table 1: Secondary Effects of SPEAR Horizontal Correctors

Corrector	Ratio $\hat{\theta}_y/\theta_x$	Secondary $\hat{\theta}_y(mrad)$	Primary $\theta_x(mrad)$	Outliers N_{ol}
HCORR1	0.0	-0.00395	0.2253	17
HCORR2	0.0	-0.00635	0.2256	17
HCORR3	-0.0651	-0.0147	0.2259	10
HCORR4	+0.0678	0.00765	0.113	7
1BB2T	0.0	-0.0116	1.106	13
2BB2T	0.0	0.0148	1.107	17
3BB2T	-0.0224	-0.0248	1.105	10
4BB2T	0.0	-0.0201	-1.106	17
5BB2T	0.0	-0.0164	0.815	14
6BB1T	0.0	-0.0150	-1.106	14
6BB2T	-0.0312	-0.0327	1.050	9
7BB1T	-0.0319	0.0352	-1.103	5
8BB1T	0.0	-0.0193	-0.948	12
9BB1T	-0.0324	0.0358	-1.104	2
10BB2T	0.0	0.0065	1.105	14
11BB2T	0.0	0.0156	-1.106	20
12BB1T	-0.0268	0.0296	-1.106	5
12BB2T	0.0	-0.00620	0.659	16
13BB1T	0.0	-0.0120	0.554	14
13BB2T	0.0	-0.0172	0.522	17
14BB1T	-0.0260	-0.0144	0.553	6
14BB2T	-0.0175	0.00973	-0.554	8
15BB1T	0.0	x	1.051	22
16BB1T	-0.0669	0.0407	-0.608	0
17BB1T	0.0	x	1.201	22
18BB1T	-0.0227	-0.0125	0.551	3

3.1 Results

The results of the analysis are summarized in tables 1 and 2. In these tables the names of the correctors, the ratio of the secondary kick angle to the primary kick angle, the kick angle in the secondary direction, the kick angle in the primary direction, and the number of outliers N_{ol} are listed. The ratio of the secondary kick angle to the primary kick angle was set to zero if it is not statistically significant, i.e. the number of outliers is greater than ten.

Eleven of the 26 horizontal correctors and eleven of the 27 vertical correctors had significant secondary effects. The proportion of the significant correctors were 42 percent.

The most significant corrector was $16BB1T$ which gave the significant t value without any outliers. Figure 1 shows the measured beam positions compared with the calculated regression. Some correctors did not become significant even after excluding the data points down to the residual three points. As an example $15BB1T$ is shown in figure 2.

The signs of the ratios of the secondary kick angle to the primary kick were negative with only three exceptions of 22 significant secondary effects. This means an upward kick of the vertical corrector has an inward kick and the inward kick of the horizontal corrector has an upward kick.

The ratios of the secondary kick angle to the primary kick ranged from 1.75 percent ($14BB2T$) to 14.7 percent ($14QDT$). The absolute values of the ratios mostly distributed in two to three percent.

3.2 Comparison with Measurement

It is not easy to measure the alignment of the correctors consisting of the trim coils of bending or quadrupole magnets. Only the independent correctors could be checked by measurement.

The rotational alignment of eight independent correctors was measured by a level meter. The result showed that four of five correctors with significant secondary effect had alignment errors in the same

Table 2: Secondary Effects of SPEAR Vertical Correctors

Corrector	Ratio $\hat{\theta}_x/\theta_y$	Secondary $\hat{\theta}_x(mrad)$	Primary $\theta_y(mrad)$	Outliers N_{ol}
VCORR1	-0.0279	0.00551	-0.1827	4
VCORR2	0.0	-0.00220	0.226	15
VCORR3	-0.0209	-0.00472	0.226	8
VCORR4	-0.0270	0.00610	-0.226	6
2QFAT	0.0	0.00478	0.1122	12
5QF1T	0.0	-0.00165	0.11	14
5QF2T	0.0	-0.00381	0.1141	20
6QF1T	0.0	-0.00349	0.0022	20
6QF2T	0.0	x	0.0022	22
7QF1T	+0.0775	-0.00853	-0.11	2
7QF2T	-0.0348	-0.00383	0.11	7
8QFAT	0.0	0.00498	0.1113	17
8QFBT	+0.0294	-0.00323	-0.11	10
11QFBT	0.0	0.00334	0.2255	13
11QFAT	0.0	0.00353	0.1113	12
12QF1T	-0.0426	-0.00473	0.111	10
12QDT	0.0	-0.00937	0.11	17
12QF2T	0.0	-0.00493	0.11	13
13QF1T	0.0	-0.00236	0.1097	12
13QF2T	-0.0676	-0.00744	0.11	5
14QF1T	-0.0601	-0.00668	0.111	1
14QDT	-0.147	-0.0166	0.113	8
15QF1T	0.0	0.00328	0.235	19
15QDT	0.0	0.00260	-0.11	22
15QF2T	-0.120	-0.0134	0.1113	5
16QF2T	0.0	0.00162	0.1113	22
17QFAT	0.0	-0.00426	0.1113	18

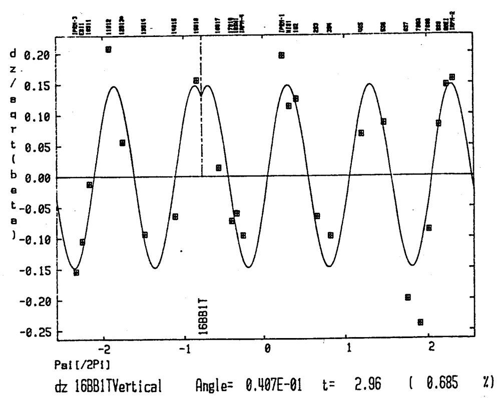

Figure 1: The measured (squares) and the calculated (curve) beam positions of the corrector with the most significant secondary kick.

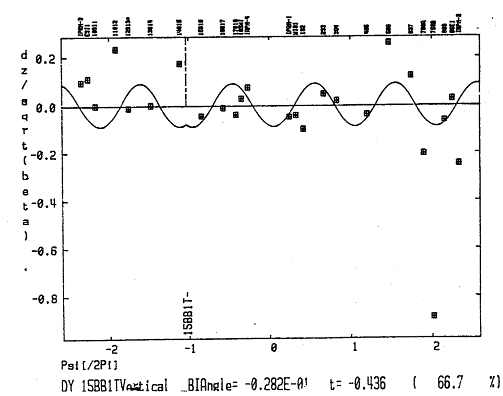

Figure 2: The measured (squares) and the calculated (curve) beam positions of the corrector with the least significant secondary kick.

Table 3: Effect of Re-alignment of Independent Correctors

Corrector	Before $\hat{\theta}/\theta_0$	Re-align	After $\hat{\theta}/\theta_0$
HCORR1	0.0	No	0.0
HCORR2	0.0	No	0.0
HCORR3	-0.0651	Yes	0.0
HCORR4	0.0678	Yes	0.0
VCORR1	-0.0279	Yes	-0.0359
VCORR2	0.0	No	0.0
VCORR3	-0.0209	Yes	0.0
VCORR4	-0.0270	No	0.0

direction with the sign of the estimated kick angle predicted. The exceptional corrector $VCORR4$ and the residual three correctors of no secondary effect were not mis-aligned.

After correcting the mis-alignments, the secondary kicks were measured and analyzed again. As shown in table 3, the secondary effects were corrected for three correctors with only an exception $VCORR1$.

3.3 Reproducibility

The reproducibility of this method was examined by applying it to the SPEAR correctors once more. Although the quantity of the secondary kick angle was not well reproduced, its statistical significance and sign were well reproduced.

4 Conclusion

A new method of analyzing the secondary effect of a corrector magnet was developed. By this method the kick angle of a secondary direction is estimated by least squares and its significance is examined by t-test combined with the outlier detection.

This method was applied to the correctors in SPEAR storage ring. The results showed that forty percent of the correctors had significant secondary effect. The independent correctors estimated to have significant secondary effects were found to be rotationally misaligned.

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