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THE USE OF GRADIENT SECTOR MAGNETS IN CHASMAN-GREEN LATTICE

S. Kato
Laboratory of Nuclear Studies
Faculty of Science, Osaka University
Toyonaka, Osaka 560, Japan

ABSTRACT

Gradient Sector magnets are proposed to use for a low emittance Chasman-Green lattice of a large synchrtron radiation ring. Formulas to represent the beam emittance are derived for the lattice. I show the gradient field make the emittance of circulating beam to be small to about half of that for the ring which use uniform bending field. The emittance is reduced by the reduction of the emittance form factor of the lattice in cooporation with the increase of the horizontal damping partition number of the betatron oscillation. The form factor and the partition number are calculated for an example. The former is represented as a function of betatron phase advance per cell.

INTRODUCTION

The design of the next generation synchrotron radiation ring is based on low emittance electron or positron storage ring with large numbers of zero dispersion straight sections for insertion devices. In some types of the proposed lattice, the Chasman-Green lattice is likely to be the most promising to be worthy to investigate for this purpose. Actually the extensive study for a large circumference storage ring using the Chasman-Green lattice has been carried out by the ESRP group[1] and the lattice is selected as a most suitable structure for the ESRP ring in the final design reprot[2]. In this study, the uniform field sector magnets were used for bending magnets.

Though a few proposal to introduce the gradient field to the bending magnet[3,4], no attepmt of analytical study exists yet to make clear the merit of it.

In this paper, I will show the gradient bending magnets in the lattice have superior performance to reduced the beam emittance in a large synchrotron radiation ring by linear calculation. The same gradient field is assumed for all bending magnet.

BEAM EMITTANCE IN THE CHASMAN-GREEN LATTICE

In this section we give how to derive the analytical expressions of the beam emittance in some detail. The horizontal beam emittance of a large electron ring is generally expressed as[5]

$$\epsilon_x = \frac{C_q \gamma^2 \theta^3 F(\psi)}{J_x} \tag{1}$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{m_e c} = 3.83\times 10^{-13}\ \mathrm{m},$$

γ= Lorentz factor of the electron beam,

θ= bending angle of a dipole magnet,

$F(\psi)$= emittance form factor as a function of the betatron phase advance per half cell ψ,

J_x= partition number of horizontal betatron oscillation.

The emittance form factor $F(\psi)$ depends on the lattice structure and is expressed as

$$F(\psi) = \frac{1}{\rho\theta^3} \langle H \rangle$$

$$= \frac{1}{\rho\theta^3}\frac{1}{\rho\theta}\int_0^{\rho\theta} (\gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2)ds \tag{2}$$

where $H=\gamma\eta^2+2\alpha\eta\eta'+\beta\eta'^2$ is the Courant-Snyder dispersion invariant, ρ is the magnetic radius of the bending magnets, α, β, γ are the Twiss parameters of the lattice and η, η' are the dispersion function and the derivative with s. The range of the integral extends only in a bending magnet. The bending angle θ is assumed to be small $\theta \ll 1$ for a large electron ring. The horizontal partition number J_x is given as

$$J_x = 1 - \frac{1}{2\pi\rho^2}\int_0^{\rho\theta} \eta(1-2n)ds \tag{3}$$

where the magnetic field index n is defined by

$$n = -\frac{\rho}{B}\frac{dB}{dx}.$$

$F(\psi)$ can be calculated by the method described by H. Wiedemann[6]. The Chasman-Green lattice is shown in Fig. 1 schematically. In the figure, 2k is the inverse

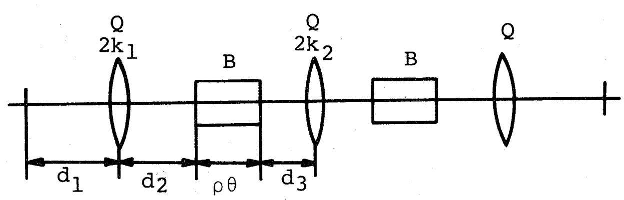

Fig. 1 Schematical Chasman-Green lattice

focal length of a focusing quadrupole magnet.

At first we assume the uniform field in the sector bending magnets. The horizontal matrix of the first half cell is

$$\begin{pmatrix} 1 & 0 & 0 \\ -k_2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & d_3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \rho\theta & \frac{\rho\theta^2}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} 1 & d_2 & 0 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2k_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

and that for the latter half cell is the matrix product in which the order of matrix of each element is reversed to the first half cell. We neglected the weak centrifugal focusing in the dipole magnet. The transfer matrix of a whole cell is generally expressed as[7]

$$\begin{pmatrix} \cos 2\psi + \hat{\alpha}\sin 2\psi & \hat{\beta}\sin 2\psi & M_{13} \\ -\frac{1+\hat{\alpha}^2}{\hat{\beta}}\sin 2\psi & \cos 2\psi - \hat{\alpha}\sin 2\psi & M_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

where $\hat{\alpha}$, $\hat{\beta}$ are the betatron function at the entrance of the cell. For the lattice of symmetrical configuration to the center, $\hat{\alpha}$ is always zero and the matrix is simplified to

$$\begin{pmatrix} \cos 2\psi & \hat{\beta}\sin 2\psi & M_{13} \\ -\frac{1}{\hat{\beta}} & \cos 2\psi & M_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

A derivative of the dispersion function $\hat{\eta}'$ is always zero at the entrance of the lattice due to the symmetrical property of the lattice. Then the dispersion function at the entrance is given by

$$\hat{\eta} = \frac{M_{13}}{1-\cos 2\psi}$$

The betatron function ψ, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\eta}$ are calculated to be

$$\sin^2\psi = \left(k_1+\frac{k_2}{2}-\frac{1}{2}k_1k_2L_1\right)(k-2k_1d_1L_1),$$

$$\hat{\beta} = \frac{1}{\sin 2\psi}(L-2k_1d_1L_1) \times\left(1-(2k_1+k_2-k_1k_2L_1)d_1-\frac{k_2L_1}{2}\right),$$

$$\hat{\gamma} = \frac{1}{\hat{\beta}},$$

$$\hat{\eta} = \frac{\theta\left(1-k_2\left(\frac{\rho\theta}{2}+d_3\right)\right)(L-2k_1d_1L_1)}{2\sin^2\psi}$$

where L is the whole length of a cell and $L_1=L-2d_1$. Then the lattice is achromatic when the relation

$$1-k_2\left(\frac{\rho\theta}{2}+d_3\right)=0 \qquad (5)$$

holds. We suppose the achromaticity of the lattice $\hat{\eta}=0$ hereafter.

Next we derive the value the values of the functions β_0 and η_0 at just inside the entrance of the bending magnet using the $\hat{\beta}$ and $\hat{\gamma}$ and the relation $\hat{\eta}=\hat{\eta}'=0$. They are obtained by the relations[8]

$$\begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{21}m_{11} & 1+2m_{12}m_{21} & -m_{12}m_{22} \\ m_{21}^2 & -2m_{22}m_{21} & m_{12}^2 \end{pmatrix} \begin{pmatrix} \hat{\beta} \\ 0 \\ \hat{\gamma} \end{pmatrix}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_0' \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

where m_{ij}'s are the elements of the transfer matrix from entrance of the cell to the entrance of the bending magnet. Then we have achromatic property between the two bending magnets

$$\eta_0=0, \quad \eta_0'=0.$$

In this case the form factor $F(\psi)$ of (2) is simplified to[9]

$$F(\psi) = \frac{\theta\rho}{20}\gamma_0 - \frac{1}{4}\alpha_0 + \frac{1}{3\rho\theta}\beta_0 .$$

For the lattice which use the gradient bending magnet, the matrix of the uniform bending magnet in (4) is replaced by

$$\begin{pmatrix} 1-\frac{K\rho^2\theta^2}{2} & \rho\theta & \frac{1}{2}\rho\theta^2 \\ -K\rho\theta & 1-\frac{K\rho^2\theta^2}{2} & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

in which K is a gradient parameter defined as

$$K = \frac{1-n}{\rho^2}$$

and we assumed

$$\left|\frac{K\rho^2\theta^2}{2}\right| \ll 1 \qquad (6)$$

to avoid a complicated calculation. The assumption is only for convenience of the calculation here and larger K values can be taken in actual cases. In these cases, the computation has to be done numerically. A negative value of K corresponds to horizontal defocacing of the magnetic field and this is in our case. The achromatic condition (5) still approximately hold for short bending magnet.

The absolute K value is limitted to hold the relation

$$-\frac{1}{2\pi}\int_0^{\rho\theta}\eta\left(\frac{1}{\rho^2}+2K\right)ds < 2$$

to guarantee the damping of energy oscillation[5]. The criterion is rather tolerant and is naturaly satisfied under the assumption (6).

Since the expression of ψ and $\hat{\beta}$ is too lengthy to write here for general case of gradient bending magnets, we write down the ψ and $\hat{\beta}$ to calculate $F(\psi)$ for the simple case where $d_1=d_2=d_3=\rho\theta=d$ for an example. In the case, the two functions are written as

$$\sin^2\psi = \left(\frac{4}{3}-4k_1d\right)(2-3k_1d) - \frac{Kd^2}{6}(7-25k_1d+18(k_1d)^2) - \frac{(Kd^2)^2}{24}(1-8k_1d)(3-4k_1d),$$

$$\frac{\hat{\beta}}{d} = \frac{1}{3\sin 2\psi}\left[-40\left(1-\frac{3}{2}k_1d\right)\left(1-\frac{6}{5}k_1d\right) + Kd^2(31-68k_1d+36(k_1d)^2) + \frac{(Kd^2)^2}{2}(7-8k_1d)(1-2k_1d)\right].$$

The parameter k_1 determine the phase advance ψ.

AN EXAMPLE

We calculate, for example, $F(\psi)$ for ρ=25 m and $\theta=\frac{\pi}{32}$ radian. We assume the ring is consisted with 32 achromat lattice. So the length of a bending magnet is $\rho\theta$= 2.45 m. For the gradient sector magnet, the gradient parameter K is taken to be $-0.05/m^2$ for example. The K value corresponds to n=32.25. The form factor $F(\psi)$ is illustrated in Fig. 2 for uniform and the gradient dipole magnets. The lower value of $F(\psi)$ for gradient magnet is apparent.

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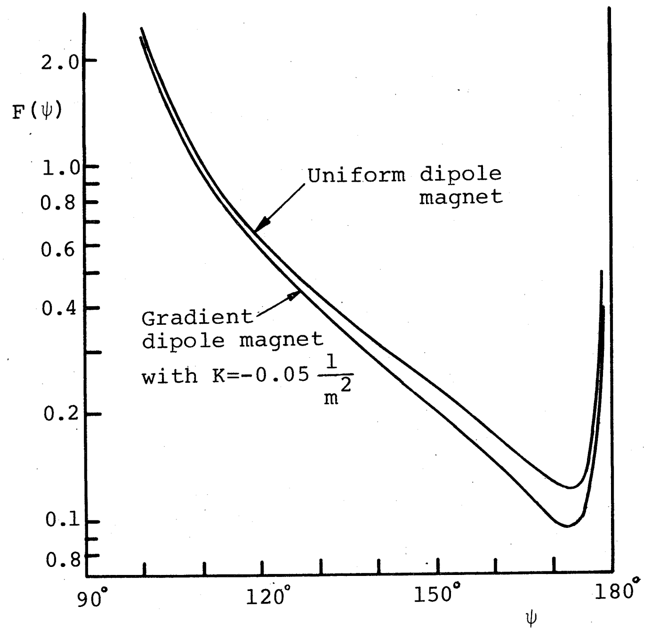

Fig. 2. Form factor F(ψ) as a function of betatron phase advance per half cell ψ.

Another factor to affect the emittance ε_x is the partition number J_x as shown in (1). The number can be written approximately in our condition as

$$J_x = 1 - \frac{K\rho^2\theta^2}{3}$$

which trun to be 1.1 in this example and generally larger than 1.0 for negative value of K. Therefore J_x has some effect to decrease the horizontal emittance in cooporation with F(ψ) for negative K value.

CONCLUDING REMARKS

I showed above the prominent performance of the Chasman-Green lattice using the gradient magnet, schematically. Because of the small electron beam emittance, a number of numerical calculations should be performed on the dynamic aperture and chromaticity as well as beam instabilities in the designing the actual ring. They are also affected by large numbers of insertion devices which contain intrinsic linear and nonlinear component of magnetic fields. We expect to make sure of sufficient large dynamic aperture due to the vertical focusing nature of the gradient fending magnet. However the low emittance naturally leads the large chromaticity. The high sextupole strengths required for chromatic correction and the consequences on dynamic aperture may be a little serious.

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