

## VARIATION OF LONGITUDINAL BEAM PROFILE DURING ACCELERATION IN TRISTAN MAIN RING

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### ABSTRACT

Variation of longitudinal beam profile is observed during acceleration in TRISTAN Main Ring (MR) using a streak camera. These are found that acceleration moves the beam center forward, that it shortens the bunch length, and that it relaxes skewness of bunch shape. The potential well distortion theory explains the phenomena.

### 1. Introduction

It is predicted by theory and verified by experiments that the potential well distortion due to the interaction of space charge with inductive surroundings lengthens the beam bunch.<sup>[1,2]</sup> The theory further tells that the potential well distortion distorts the longitudinal Gaussian distribution. In this work the longitudinal beam distribution is measured as a function of beam energy by a streak camera. Bunch center, bunch length and bunch skewness are experimentally calculated as measures of the distribution. They are compared with the calculation based on the potential well distortion model.

### 2. Longitudinal Profile Measurement

All data are taken on July 1987 at runs for physics experiments where two electrons bunches collide with two positron bunches. The data presented here are derived from synchrotron radiation of electron bunches. The TRISTAN MR has 20 klystrons, each feeds its rf into 4 cavity structures. So the number of rf structures is 80. Two bunches of electrons with energy of 7.4GeV are injected to the ring where two bunches of positrons, each has intensity of approximately 2.5mA, already exist. The bunches are then accelerated upto 26GeV. The beam intensity at the beginning of the acceleration is approximately 9mA in total of four bunches, two electrons' and two positrons'. The beam loss during the acceleration is not serious. The intensity of the wiggler magnet is kept constant till the beam energy reaches 11.6GeV, and it is then decreased linearly to zero at 18.8GeV. The rf voltage is changed so as to keep the synchrotron tune approximately constant during the acceleration.

The streak camera used for the longitudinal profile measurement is Hamamatsu C1587 with M1955 synchro-scan unit and M2887 dual timebase unit. The shutter of the camera is

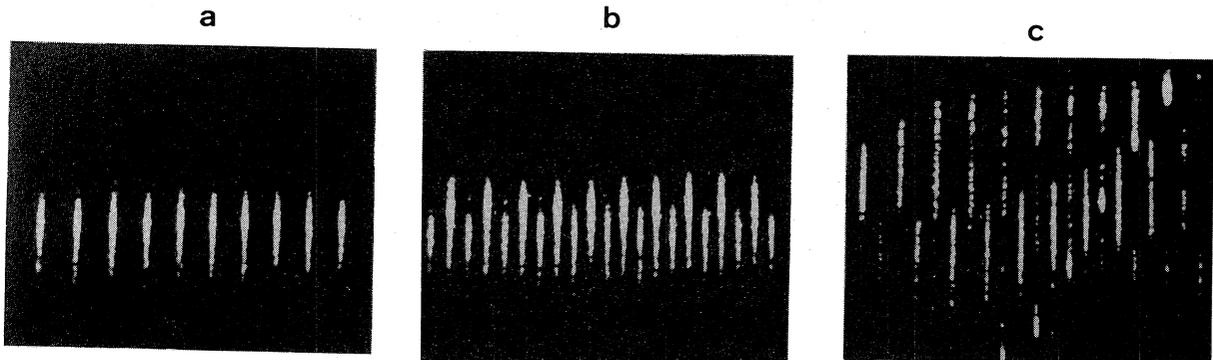


Fig. 1 Examples of streak pictures. Full scales are horizontally  $10\mu\text{sec}$  and vertically  $568\text{psec}$ . (a) one bunch case. (b) two bunches case with different intensities. (c) two bunches case with instability.

triggered by a clock synchronous with the revolution. Once triggered, photoelectrons are vertically deflected in the camera by sine wave whose frequency is 1/4 of that of the cavities. Besides the usual pair of vertical deflectors, it has another pair which scans the streak image horizontally. We can thus observe the streak image of each revolution, or the image of each bunch of each revolution. Time resolution of the streak camera is 6psec in the present operation. Fig. 1 shows examples of streak pictures: Fig. 1 (a) is for the case where only one electron bunch exists in the ring, while (b) is for the case where two electron bunches with different intensities exist. Note that the one with more current is longer, and its position leads the other. Fig. 1 (c) is for the case an instability is occurring.

A streak picture is digitized into 256 by 256 elements. Intensity of each element is digitized into 8bits. Each picture contains 10 (single bunch case) or 20 (two bunches case) traces. Data of each trace is horizontally integrated to give one dimensional longitudinal distribution  $y_i$ . Following quantities are then calculated;

$$\begin{aligned}\bar{x} &= (1/n) \sum y_i x_i, \\ \sigma^2 &= m_2 = (1/n) \sum y_i (x_i - \bar{x})^2, \\ m_3 &= (1/n) \sum y_i (x_i - \bar{x})^3, \\ s &= m_3 / m_2^{2/3},\end{aligned}\quad (2.1)$$

where  $n = \sum y_i$ . The quantity  $\bar{x}$  characterizes the bunch position. The quantity  $\sigma$  is nothing but the bunch length. The last quantity  $s$  is called skewness in statistics, which we use here as a measure of distortion of the Gaussian distribution. It has no dimension, and is zero when the distribution is perfectly symmetric. Statistical errors of the experimental data are calculated from traces on a single streak picture.

### 3. Potential Well Distortion

The longitudinal current distribution is given by

$$I_n(t) = K \exp\left(-\frac{t^2}{2\sigma^2} - \frac{G}{\sigma^3} \int_0^\infty s(\tau) I_n(t-\tau) d\tau\right), \quad (3.1)$$

where  $I_n(t) = I(t)/I_p$  and  $I_p$  is the peak current of the unperturbed Gaussian bunch. The constant  $K$  is chosen so as to conserve the total charge;

$$\int_{-\infty}^{+\infty} I_n(t) dt = \sqrt{2\pi}\sigma. \quad (3.2)$$

The natural bunch length  $\sigma$  depends both rf operation and wiggler operation, which will be shown later together with the experimental results. Finally,  $G = \alpha I / (f_0^2 \nu_s^2 E (2\pi)^{5/2})$  is the scaling parameter of the strength of the potential well distortion, where  $f_0$  denotes the revolution frequency.

The function  $s(\tau)$  is the integrated wake function. It gives the response at time  $\tau$  to a unit current step at  $\tau$ , which is calculated with the optical resonator model for each rf cavity.<sup>[3]</sup>

$$\begin{aligned}s(\tau) &= \frac{1}{2} \sum_{n=1}^{\hat{n}} \left(\frac{R}{Q}\right)_n \sin(\omega_n \tau) \\ &+ C_{sv} \int_0^x \left[ \frac{\pi}{4} (1+4x) \exp(2x) \operatorname{erfc}(\sqrt{2x}) - \sqrt{\frac{\pi x}{2}} \right] dx \\ &- 2C_{sv} \int_0^s \frac{(u+1) \sin xu^2}{u(u^2+2u+2)^2} du,\end{aligned}\quad (3.3)$$

where  $x = \omega_{sv} \tau$ ,  $u^2 = \nu$ ,  $\nu = \omega/\omega_{sv}$  and  $s = \sqrt{\hat{n}} = \sqrt{\hat{\omega}/\omega_{sv}}$ . The first term gives the truncated wake. The calculation of  $R/Q$  is found elsewhere.<sup>[4]</sup> We set  $\hat{n} = 59$  in this calculation. The second and third terms complement the missing part due to the truncation. The quantities  $C_{sv}$  and  $\omega_{sv}$ , which are called Sessler-Vainstein constant and Sessler-Vainstein frequency, are 650ohm and 7.715GHz, respectively. Fig. 2 shows the integrated wake function, or the step response function, used in the present calculation.

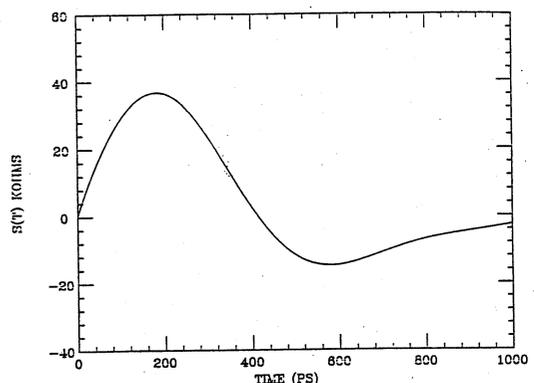


Fig. 2 The wake function (a) and its integration (b) used for the calculation.

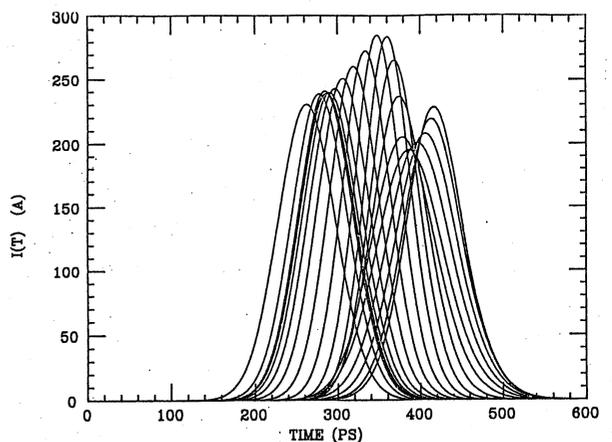


Fig. 3 Calculated longitudinal profiles at various beam energy. The energy ranges from 8GeV (rightest curve) to 26GeV (leftest) in 1 GeV step.

#### 4. Results and Discussions

Fig. 3 gives calculated longitudinal profiles at various beam energy. The profile is not very symmetric at lower energy region, which approaches Gaussian as the energy rises. It is sharpest at the energy where the loss parameter gets maximum, which is a measure of the total energy lost to cavities expressed in volt per unit charge. Experimentally, the beam gets unstable in this energy region, as shown in Fig.1(c), where it is impossible to obtain the parameters of eq.(2.1).

Fig. 4 compares the experimental results with calculation. The dash curve in Fig.4(a) shows the position decided solely by the synchronous rf phase, or the position of the bunch with null intensity. The bunch position shifts toward a crest of rf field due to higher order mode losses at the finite bunch current. The experimental data show twice as a large shift as the calculation. In Fig.4(b), the calculation tells that the bunch length gets short as the wiggler is weakened, and it recovers at higher beam energy. It agrees qualitatively with the experiment. The experimental profile skews more strongly than the calculated at lower energy in Fig.4(c).

The calculation of the wake function  $s(\tau)$  takes into account only the longitudinal normal mode of a cavity. The wake field of the vacuum chamber is not included. It is however known that the longitudinal impedance of the vacuum chamber increases a higher order mode loss, if the bunches are short.<sup>[8]</sup> In other words, bunch shift and profile distortion would get larger with the vacuum chamber wakefield. It must be even possible to derive the vacuum chamber wakefield from the difference between the calculation and the experiment. The present result suggests that the loss parameter of vacuum chambers is comparable with that of rf cavities in the region of 30psec or 1cm bunch length.

#### ACKNOWLEDGEMENTS

We would like to thank all the members of TRISTAN operation group for their useful discussion.

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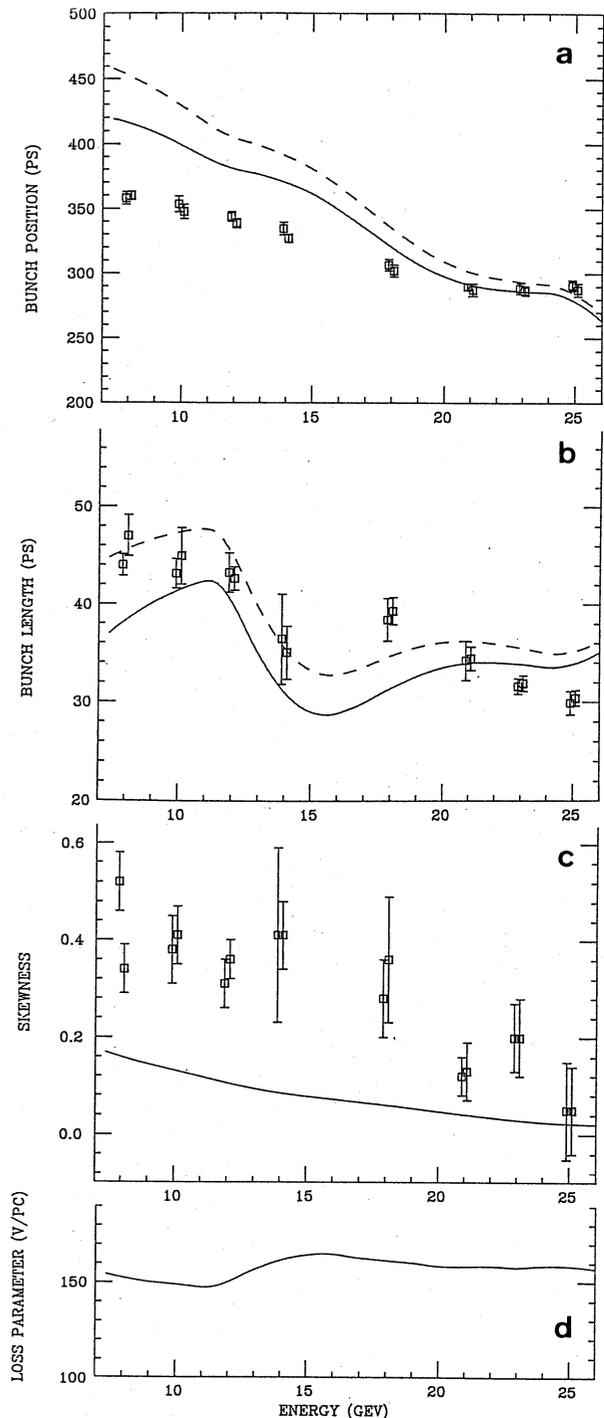


Fig. 4 Bunch position (a), length (b) skewness (c) and loss parameter (d) as a function of beam energy. The dashed curve in (a) shows the position corresponding to the synchronous rf phase. The dashed curve in (b) shows the natural bunch length. The pairs of experimental data correspond to the pairs of electron bunches.