A METHOD TO MEASURE TWO-DIMENSIONAL MOTION OF ORBIT CENTER
WITH THREE RADIAL DIFFERENTIAL PROBES

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INTRODUCTION

It quite facilitates the tuning of acceleration oribt, e.g. centering and off-centering of the beam, to measure the positions of orbit center in both x and y directions. Such measurement necessitates, in principle, at least three radial differential probes. As shown in plan view of Fig. 1, in our SSC the radial differential probes are arranged at three azimuthal positions of sector-edge lines. We describe below the method of measurement of the position of orbit center with these probes, which is the extension of the VICKSI's method[1].

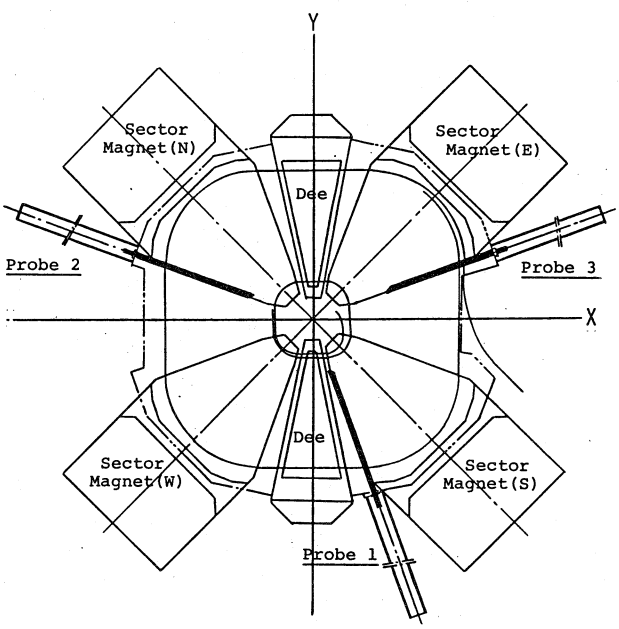

Fig. 1. Arrangement of radial differential probes in the SSC.

ACCELERATION EQUILIBRIUM ORBIT (AEO)

We define the valley mid-line and the dee mid-line as x- and y- axes, respectively. In the preceding paper[2] we presented the condition for the acceleration equilibrium orbit (AEO). This orbit is depicted in Fig. 2. The radial gain Δr is the difference in radii between the successive AEO's in one turn. The angles, a and z, come from the above condition. These are given by

$$a = \frac{\Delta r}{4\sin(\pi\nu_r/2)}\left(\frac{r r_o'}{r_o}\right), \tag{1}$$

$$z = a\cdot\cos\left(\frac{\pi}{2}\nu_r\right), \tag{2}$$

where r_o (radial displacement) and r_o' (angle) represent the principal axes of the eigen-ellipse at the radius of r on the x-axis, and ν_r the corresponding radial focusing frequency. From Fig. 2 the radii of the AEO at each azimuth of probe, P_{e1} - P_{e6}, are given by

$$P_{e1}=\frac{1}{c}(1+zt)(r-\frac{1}{4}\Delta r)=\frac{1}{c}(r-\frac{1}{4}\Delta r), \tag{3}$$

$$P_{e2}=\frac{1}{c}(1+zt)(r+\frac{3}{4}\Delta r)=\frac{1}{c}(r+\frac{3}{4}\Delta r), \tag{4}$$

$$P_{e3}=\frac{1}{c}(1+at)r=\frac{1}{c}(r+\frac{1}{4}tf\Delta r), \tag{5}$$

$$P_{e4}=\frac{1}{c}(1+at)(r+\Delta r)=\frac{1}{c}\{r+(1+\frac{1}{4}tf)\Delta r\}, \tag{6}$$

$$P_{e5}=\frac{1}{c}(1-at)(r+\frac{1}{2}\Delta r)=\frac{1}{c}\{r+(\frac{1}{2}-\frac{1}{4}tf)\Delta r\}, \tag{7}$$

$$P_{e6}=\frac{1}{c}(1-at)(r+\frac{3}{2}\Delta r)=\frac{1}{c}\{r+(\frac{3}{2}-\frac{1}{4}tf)\Delta r\}, \tag{8}$$

where $c=\cos\alpha$, $t=\tan\alpha$ and

$$f = \frac{1}{\sin(\pi\nu_r/2)}\left(\frac{r r_o'}{r_o}\right). \tag{9}$$

Because the value of z is negligible, it is taken to be null in eqs. (3) and (4). We also neglect the term, $(\Delta r)^2$, in eqs. (5)-(8).

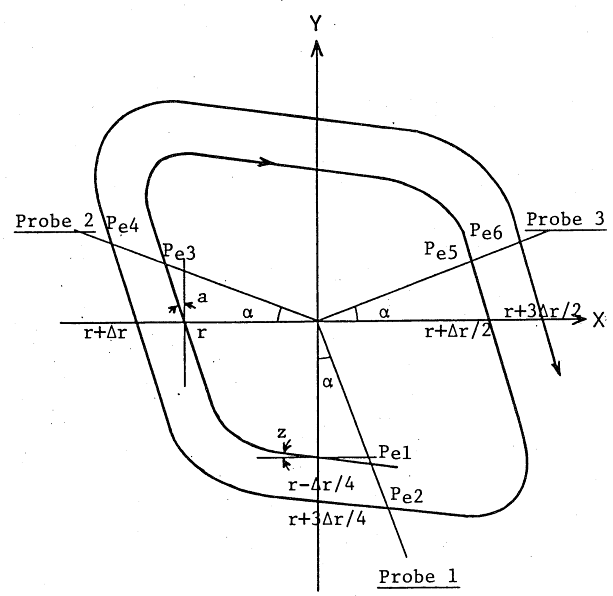

Fig. 2. Acceleration equilibrium orbit (AEO) and configuration of three radial differential probes. In case of our SSC, $\alpha = 20^\circ$.

ORBIT CENTER

Fig. 3 shows the beam trajectory with respect to the AEO; the orbit center existing at the coordinate of (A_x, A_y). We neglect the rotation of the orbit center during about $1\frac{1}{2}$ turns. From Fig. 3 the radius (P_1) of the position where the probe 1 catches the beam is given by

$$P_1 = P_{e1} + \frac{1}{c}(tA_x - A_y). \tag{10}$$

In the same way,

$$P_2 = P_{e2} + \frac{1}{c}(tA_x - A_y). \tag{11}$$

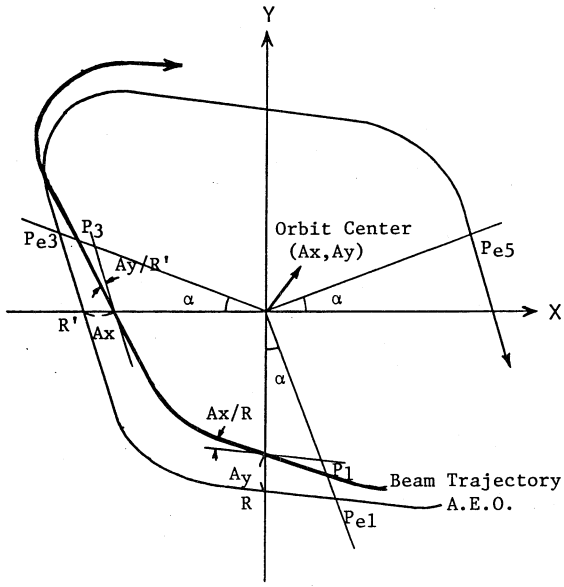

Fig. 3. Beam trajectory with respect to the AEO, the orbit center at the radius of R on the x-axis being given by (A_x, A_y).

$$P_3 = P_{e3} - \frac{1}{c}(A_x - tA_y), \tag{12}$$

$$P_4 = P_{e4} - \frac{1}{c}(A_x - tA_y), \tag{13}$$

$$P_5 = P_{e5} + \frac{1}{c}(A_x + tA_y), \tag{14}$$

$$P_6 = P_{e6} + \frac{1}{c}(A_x + tA_y). \tag{15}$$

From eqs. (3)-(15) the following relations are obtained.

$$A_x = \frac{1}{8}\cos\alpha\,(P_6 + 3P_5 - 3P_4 - P_3) + \frac{1}{8}\sin\alpha f\,(P_6 - P_5 + P_4 - P_3), \tag{16}$$

$$A_y = \frac{1}{2}\cdot\frac{1}{1+\tan\alpha}\{2A_x\tan\alpha - \cos\alpha\,(P_1 + P_2 - P_3 - P_5)\}. \tag{17}$$

VERIFICATION OF THE METHOD BY COMPUTER SIMULATION

In order to verify the reliability of these formulae, we simulated this three-probe measurement of the orbit center as well as the four-probe measurement; the four probes being arranged on the x- and y-axes. The calcualtion was made for $^{12}C^{6+}$ with injection energy of 50.8 MeV. The first harmonic field of 15 G was applied. The value of f changes from 1.31 to 1.7 with the radius. In the calculation it was fixed to be 1.5. Fig. 4 shows the results a) in the first fifteen turns and b) in the last seven turns obtained by the three (solid line) and four (dashed line) probe measurements.

It was found that the three-probe measurement mentioned here is quite reliable in tracing the two-dimensional motion of the orbit center.

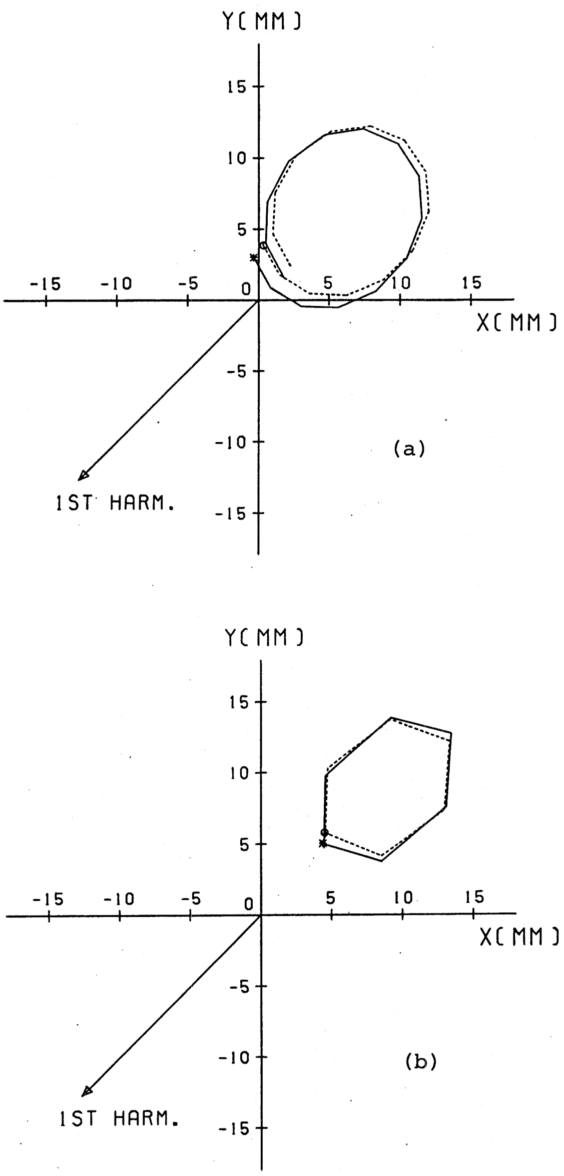

Fig. 4. Comparison of the three (solid line) and four (dashed line) probe measurements. a) in the first fifteen turns (injection region). b) in the last seven turns (extraction region).

References

1. W.M.Schulte, The Thoery of Accelerated Particles in AVF Cyclotrons, thesis, Eindhoven University of Tecnology (1978).
2. A.Goto et al., Proc. of the 9th Int. Conf. on Cyclotrons and their Applications, Caen, (1983) 439.