THE STUDY OF NEW TYPE OF RF ACCELERATION IN SCALING FFAG ACCELERATOR

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Abstract

High power proton driver to produce intense secondary particle beams is required for various fields. Fixed-field alternating gradient (FFAG) accelerator is one of the possible candidates for such proton drivers. In order to produce much higher intensity proton beams in scaling type of FFAG, a new type of acceleration scheme, called serpentine acceleration, is considered in this paper. The longitudinal hamiltonian for serpentine acceleration is derived analytically. The application for proton driver, based on serpentine acceleration, is also presented.

FEATURES OF SCALING FFAG

In cylindrical coordinates, the magnetic field in scaling FFAG is given by

\[ B_z(r, z = 0) = B_0 \left( \frac{r}{r_0} \right)^k, \]

(1)

where \( r \) is the radial coordinate with respect to the center of the ring. \( B_0 \) is the magnetic field at the reference radius \( r_0 \). \( k \) is the geometric field index and \( z \) is the vertical coordinate. Magnetic fields of the scaling FFAG magnets are kept constant during particle acceleration. For this reason, rapid acceleration with high repetition rate is possible. Also zero chromaticity is achieved by using non-linear magnetic field which is expressed by Eq.(1). It makes a betatron tune constant even if particle momentum changes. Therefore stable beam acceleration can be realized.

SERPENTINE ACCELERATION IN SCALING FFAG

The principle of serpentine acceleration is to accelerate a beam between stationary buckets and a beam passes through the transition energy during acceleration. The constant rf frequency is adopted in the serpentine acceleration. Therefore cw operation can be achieved.

Longitudinal Hamiltonian in scaling FFAG

In order to examine features of serpentine acceleration, longitudinal hamiltonian for the stationary bucket acceleration in scaling FFAG is derived analytically.
The closed orbits for different momentum \( P \) is given by

\[
r = r_0 \left( \frac{P}{P_0} \right)^{\frac{1}{k+1}},
\]

where \( P_0 \) is the reference momentum at \( r_0 \).

With constant rf frequency in the scaling FFAG, the longitudinal phase discrepancy per revolution \( \Delta \phi \) is written by

\[
\Delta \phi = 2\pi (f_{rf} \cdot T - h),
\]

where \( h \) is the harmonic number, \( f_{rf} \) is the rf frequency and \( T \) is the revolution period of a non-synchronous particle. Equation (3) becomes

\[
\Delta \phi = \frac{hT}{T_s} - h,
\]

where \( T_s \) is the revolution period of a synchronous particle. Equation (4) is also expressed with another description based on Eq.(2) as follows;

\[
\frac{T}{T_s} = \left( \frac{r}{r_s} \right)^\alpha \frac{P/E}{P_s/E_s},
\]

where \( r_s \) is the reference radius, \( \alpha \) is the momentum compaction factor and \( E_s \) is the reference energy at the reference radius. As shown in Fig.1, the two stationary energies, \( E_{s1} \) and \( E_{s2} \), close to each other when rf frequency is near the revolution time of transition energy. Combining Eq.(4) and Eq.(5), the phase difference becomes

\[
\Delta \phi = 2\pi h \left( \frac{P^{1-\alpha}}{E_s} E^{\alpha-1} - 1 \right). \tag{6}
\]

Now we exchange \( \Delta \phi/2\pi \) and \( d\phi/d\theta \) to derive the phase and energy equation of longitudinal motion,

\[
\frac{d\phi}{d\theta} = h \left( \frac{P^{1-\alpha}}{E_s} E^{\alpha-1} - 1 \right), \tag{7}
\]

\[
\frac{dE}{d\theta} = \frac{eV_{rf}}{2\pi} \sin \phi, \tag{8}
\]

where \( V_{rf} \) is the rf voltage per turn and \( \theta \) is an azimuthal angle in the machine. We introduce the energy variable \( E \) canonically conjugate to the coordinate variable \( \phi \). Equation(7) and (8) derive the longitudinal hamiltonian:

\[
H(E, \phi; \theta) = h \left( \frac{1}{\alpha + 1} \frac{P^{\alpha+1}}{E_s P_s^{\alpha+1}} - E \right) + \frac{eV_{rf}}{2\pi} \cos \phi. \tag{9}
\]

**Longitudinal Phase Space**

The features of serpentine acceleration are examined by hamiltonian (Eq.(9)). As shown in Fig.2, when the two stationary energies are far from each other, the two stationary buckets are also separated. When the two stationary energies close to each other, however, a channel between the two stationary buckets appears as shown in Fig.3. If particles can be accelerated using this channel, total energy gain through the acceleration becomes larger than the total energy gain within a stationary bucket.

**Minimum rf Voltage**

The minimum rf voltage to make serpentine acceleration scheme is derived from Eq.(9). Since the separatrix goes through two unstable fixed points as shown in Fig.4, the relation between \( H(E_{s1}, \pi) \) and \( H(E_{s2}, 0) \) is

\[
H(E_{s1}, \pi) = H(E_{s2}, 0). \tag{10}
\]

The relation between \( E_{s1} \) and \( E_{s2} \), which are shown in Fig.1, is

\[
E_{s1} P_s^{\alpha-1} = E_{s2} P_s^{\alpha-1}. \tag{11}
\]
From Eq.(10) and Eq.(11), the minimum rf voltage to make serpentine acceleration is derived;

$$V_{rf} = \pi h \left[ \frac{1}{\alpha + 1} \left( \frac{P_{s1}^2}{E_{s1}} - \frac{P_{s2}^2}{E_{s2}} \right) + (E_{s2} - E_{s1}) \right].$$  \hspace{1cm} (12)

Equation(12) shows that once the $k$ value and the stationary energy are given, the minimum rf voltage to achieve serpentine acceleration can be calculated.

### APPLICATION

Proton driver is considered with serpentine acceleration. Parameters of proton driver are summarized in Table 1. The beam trajectories in the longitudinal phase space are shown in Fig.5. As shown in Fig.5, injection kinetic energy is $500 \text{ MeV} \pm 1\%$. Injection phase range is from 0.25 to 0.3 $2\pi \text{ rad}$. Final kinetic energy is $2018 \text{ MeV} \pm 1.4\%$. The number of turn during acceleration is 35 turns. The ratio of injection to final momentum is 2.6. Furthermore, the injected beam shape is rotated in phase space.

<table>
<thead>
<tr>
<th>Table 1: The parameters of proton accelerator</th>
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<tr>
<td>Stationary kinetic energy</td>
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<tr>
<td>Mean radius ($\gamma_s$)</td>
</tr>
<tr>
<td>$k$ value</td>
</tr>
<tr>
<td>Harmonic number h</td>
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<tr>
<td>rf voltage/turn</td>
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<tr>
<td>rf frequency</td>
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### CONCLUSION

In order to obtain a high power proton beam with high repetition rate, serpentine acceleration has been proposed for the scaling FFAG. The longitudinal hamiltonian in the scaling FFAG has been obtained analytically. By using longitudinal hamiltonian, the features of serpentine acceleration have been examined. For further study of serpentine acceleration in scaling FFAG, some experiments with real machine should be done.

### REFERENCES