Electron Cloud Instability in SuperKEKB Low Energy Ring

Yoshiaki Susaki*, Kazuhito Ohmi†, KEK-ACCL, Oho, Tsukuba, Ibaraki, Japan

Abstract

We study the issue of coherent instabilities due to electron clouds by numerical simulations for SuperKEKB. We first calculate electron cloud density by simulating the motions of the electrons emitted from the chamber wall. By introducing an ante-chamber we can reduce the number of electrons emitted from the chamber wall. We evaluate the relation of the electron density and the efficiency of the ante-chamber. Next we study a perturbation to the beam motion (bunch by bunch wake field) and the growth rate of the coupled bunch instability. From those studies we estimate the effective value of quantum efficiency safe for avoiding coherent instabilities. Finally the threshold of the electron cloud density for the stability is estimated for SuperKEKB by single bunch numerical simulations.

INTRODUCTION

Coherent instabilities caused by the interaction between positron bunches and electron clouds is one of the serious issues [1] that must be avoided for low emittance rings to be operated steadily. Let us begin with the process of how an electron cloud is built up. At first positron beam emits photons by synchrotron radiation. Then electrons are produced at the chamber wall by photoemission. The electrons are attracted and interact with the positron beams and hit the chamber wall after several 10 ns. The electrons are absorbed in the chamber wall or yield secondary electrons that hit the chamber wall after several 10 ns. The electrons are absorbed in the chamber wall or yield secondary electrons according to their energy. When electrons are supplied continuously by multi-bunch operation with a narrow spacing, they accumulate in the chamber. Consequently the electron cloud is built up.

The number of photons emitted by one positron is given by

\[ N_{\gamma} = \frac{5\pi}{\sqrt{3}} \alpha \gamma, \]

where \( \alpha \) and \( \gamma \) are the fine structure constant and the Lorentz factor, respectively. In the case of SuperKEKB-LER the number of photons per unit meter is \( Y_{\gamma} = 0.17m^{-1} \). The quantum efficiency for photoelectrons is considered around \( \eta = 0.1 \). Thus the number of electrons produced by one positron per unit meter becomes \( Y_{p,e} = 0.017m^{-1} \). The bunch population for SuperKEKB-LER is designed to be \( N_p \sim 10^{11} \). Then the number of electrons produced by one bunch per unit meter is given by \( Y_{p,e}N_p = 1.7 \times 10^{9}m^{-1} \). On the other hand, the maximum secondary emission yield \( \delta_{Z,\text{max}} \) is known to be \( 1.0 \sim 1.2 \).

Next we proceed to the introduction of coherent instabilities caused by the accumulated electrons. As a bunch passes through the electron cloud, the remnants of the position of the bunch is left behind in it and the part of the electron cloud oscillates. Coherent instabilities can occur when there are resonance between the oscillation modes of the electron cloud and those of backward bunches.

There are two kinds of coherent instabilities. One is known as coupled bunch instability (CBI), which is caused by the correlation among bunches through the oscillation of an electron cloud. The threshold for CBI is determined by some damping effects. The other is single bunch instability (SBI), which is caused by the correlation among positrons within a single bunch. SBI is considered to be caused essentially by a head-tail motion in a bunch [2]. The coherence of the transverse oscillation is weakened by the longitudinal oscillation associated with momentum compaction, which is known as Landau damping. A stability condition for SBI are determined by the balance of the growth of the beam and Landau damping.

Remarkably the threshold of SBI depends only on a local electron density. From the stability condition the threshold of the electron cloud density is given by

\[ \rho_{e,\text{th}} = \frac{2\gamma\nu_\epsilon\sigma_z/c}{\sqrt{3}KQr_03L}, \quad Q = \min(Q_{nl}, \omega_e\sigma_z/c), \quad (1) \]

where \( \nu_\epsilon, \omega_e, \sigma_z, c, Q, r_0, \beta, L \) are the synchrotron tune, the angular oscillation frequency of the electrons, the size of the bunch in the longitudinal direction, the speed of light, the classical electron radius, a beta function, the circumference of a ring, respectively. \( K \) is such a quantity as characterizes cloud size effect and pinching. For the value of \( Q \), we choose the smaller one of \( Q_{nl} \) and \( \omega_e\sigma_z/c \). We use \( K = \omega_e\sigma_z/c \) and \( Q_{nl} = 7 \) [3] for analytic estimations. For SuperKEKB \( \rho_{e,\text{th}} \) becomes \( 1.1 \times 10^{11}m^{-3} \).

One of the methods of reducing the electron cloud density is changing the form of the chamber wall. For instance, we can introduce an ante-chamber. With the ante-chamber it becomes hard for the electric field from the positron bunches to affect the electrons emitted from the chamber wall since electric field can not enter the chamber slot.

Our purpose here is to estimate an electron density which is safe for avoiding the coherent instabilities by numerical simulations for the SuperKEKB. We first calculate electron cloud densities by simulating the motions of the electrons emitted from the chamber wall. Then we evaluate it in relation with the efficiency of the ante-chamber. Next we study a perturbation to the beam motion and the growth rate of CBI. Finally we determine the threshold of the electron cloud density for the case of SBI by numerical simulations using particle in cell method (PIC) [4].
NUMERICAL SIMULATIONS

Analysis of electron cloud density

We first calculate the electron density by simulating the motion of electrons emitted from the wall of the cylindrical chamber with the radius of 48 mm. In Figure 1 we see that the density increases and saturates as bunches pass. With an ante-chamber we can reduce the number of the electrons emitted from the chamber. The efficiency of the ante-chamber may be naturally translated to the value of effective quantum efficiency with it. In Figure 2 we plot the electron densities $\rho_e$ against the effective quantum efficiencies $\eta$. Remember that the analytical value of the threshold of the electron cloud density for SBI for Super KEKB is $1.1 \times 10^{13} \text{m}^{-3}$. The curve representing the density near the beam in Figure 4 implicitly lead us to conclude that the quantum efficiency rate needs to be reduced to 0.001 at least in order to prevent SBI.

Figure 1: Increase of electron density for $\delta = 1.2$ and $\eta = 0.1$ plotted against quantum efficiencies

The same simulation is performed for the case with the antechamber. In Figure 3 we compare the distribution of electrons in the ring with the ante-chamber and one with the cylindrical chamber. By setting $\delta_{2,\text{max}}$ zero, we assume here that no secondary electrons are produced, so that we can evaluate the efficiency of the antechamber rather directly. We calculate the ratio of the densities at the beam pipe of the ante-chamber and the cylindrical chamber. In Figure 4 we plot the ratio of average electron densities with the ante-chamber and one with the cylindrical chamber. By setting $\delta_{2,\text{max}} = 0$ we find that the antechamber can reduce $\eta$ in 3 percent effectively. In fact this is not sufficient for keeping the electron density below the threshold. The actual $\eta$ is, however, expected to be reduced to 0.001 together with solenoid magnets.

Figure 2: Electron densities density for $\delta = 1.2$ and $\eta = 0.1$ plotted against quantum efficiencies

$\Delta y$ be the vertical displacement of a positron in the bunch. Then the equation of the motion of $\Delta y$ is given by

$$\frac{d^2 y(t)}{dt^2} + \omega_\beta^2 y(t) = -\frac{N_{\text{e}}\gamma}{N_0} \sum_{n=1}^{\infty} \frac{\Delta \eta y(\pm -nT_0/h)}{\Delta y \gamma T_0},$$

where $\omega_\beta$, $T_0$ and $h$ are the betatron oscillation number, the revolution time and the harmonic number, respectively. The index $n$ denotes a bunch which is the $n$th ahead of the 0th one. We assume that each bunch consists of $N_{\text{e}}$ positrons and produces $N_{\text{e}}\gamma$ electrons during one revolution.

We define a mode number $m$ and its frequency $\Omega_m$ as follows:

$$y_n^{(m)}(t) = e^{2\pi imn/h} y_0^{(m)}(t)$$

$$y_n^{(m)}(t) = \hat{y}_n^{(m)} e^{-i \Omega_m t}.$$  

Then we obtain a dispersion relation;

$$\Omega_m - \omega_\beta = \frac{i}{4\pi \gamma \nu_y} \frac{N_{\text{e}}\gamma}{N_0} \sum_{n=1}^{\infty} \frac{d \hat{y}_n^{(m)}}{dy} \left( \frac{-nT_0}{h} \right) e^{2\pi im(n + \nu_y)/h}.$$  

(5)

The growth rate of CBI is given by the imaginary part of $\Omega_m$. Once momentum kicks to the electrons are obtained, we can calculate growth rates associated with each mode by using this equation. The result is shown in the right plot in Figure 5.
In Figure 6 we plot the growth rate associated with the unstable modes for various effective values of $\eta$. We see that the growth rate is suppressed as the effective value of $\eta$ is reduced. The growth rate turns out to be 0.02 for $\eta = 0.001$. From the empirical point of view, this figure of the growth rate is not so severe that the growth could be suppressed with feedback system. It should be remembered that this value of $\eta$ corresponds to the threshold of SBI when $\eta$ is evaluated as the function of the electron density. This implies that CBI could be circumvented below the threshold of SBI by utilizing the feedback.

![Figure 5: Momentum kicks to the electron cloud (left) and growth rate associated with each mode (right) for $\eta = 0.001$](image)

![Figure 6: The growth rate as a function of $\eta$](image)

**Analysis of single bunch instability**

Finally we study SBI by numerical simulations with PIC. Electron clouds are put at several positions in the ring. Beam-cloud interaction is calculated by solving two-dimensional Poisson equation on the transverse plane. A bunch is sliced into 20-30 pieces along the longitudinal direction. Note that the number of the cells are large enough for describing the oscillation of the beam.

Figure 7 shows the profiles of the beam size obtained by the simulation for SuperKEKB. Both the result without dispersion ($\eta_y = 0.0$) and the one with ($\eta_y = 0.2$) are presented. From these plots we estimate that the threshold of the electron density is $2.4 \times 10^{11} m^{-3}$. With dispersion the threshold becomes lower and its value is $2.2 \times 10^{11} m^{-3}$. Let us see if there are coherent motions above the threshold. In Figure 8 we show the bunch and the electron cloud profiles at 4000 turn. We observe that above the threshold there is coherence between the bunch position and the center of mass of the electron cloud. On the other hand there is no coherent motion below the threshold.

![Figure 7: The profile of the beam size without dispersion $\eta_y = 0.0$ (left) and with dispersion $\eta_y = 0.2$ (right)](image)

![Figure 8: The profiles of the bunch and the electron cloud at 4000 turn above (left) and below (right) the threshold of SBI](image)

**CONCLUSIONS**

We performed the numerical simulations for SuperKEKB and estimated the value of electron density safe for overcoming coherent instabilities. We conclude that the effective value of quantum efficiency $\eta$ should be reduced to 0.001 by using the ante-chamber in order to keep the electron density below the threshold of SBI. However, the antechamber alone seems not to be sufficient for achieving this value, but together with solenoid it is expected to cure the situation. From the analysis of the momentum kicks to the electron cloud, the growth rate of CBI at $\eta = 0.001$ turns out to be not so severe that could be controlled with the feedback. We thus expect that CBI could be tamed below the threshold of SBI.

From the single bunch numerical simulation the threshold of the electron cloud density for the stability has been estimated for SuperKEKB.

**REFERENCES**