

# A STUDY OF BEAM LOADING EFFECT AND ITS COMPENSATION ON ALTERNATE PERIODIC STRUCTURE CAVITY ON E-DRIVEN ILC POSITRON SOURCE

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## Abstract

In E-Driven positron source of ILC, the generated positron is captured by RF accelerator by APS cavity. The positron is initially placed at the deceleration phase and gradually slipped down to acceleration phase. Because the beam-loading is expected to be more than 1A in a multi-bunch format with a gap, the compensation is essential to obtain uniform intensity over the pulse. A conventional method for the compensation controlling the timing doesn't work because RF and Beam induced field are in different phase. In this manuscript, we discuss the compensation with a finite phase difference. As a conclusion, a simple amplitude and phase modulation on the input RF is a solution.

## INTRODUCTION

ILC is an e+e- linear collider with CME 250 GeV - 1000 TeV [1]. It employs Super-conducting accelerator (SCA) to boost up the beam up to the designed energy. The beam is accelerated in a macro pulse with 1300 bunches by 5 Hz repetition. The bunch charge is 3.2 nC resulting the average beam current 21  $\mu$ A. This is a technical challenge, because the amount of positron per second is more than 40 times larger than that in SLC [2].

The configuration is schematically shown in Figure 1. The generated positron is captured and boosted up to 5 GeV by the capture linac and positron booster. In the E-Driven ILC positron source, 3.0 GeV electron beam is the driver for positron generation with 16 mm W-Re alloy target. The 16 mm W-Re target is rotating with 5.0 m/s tangential speed to prevent a potential target damage. FC (Flux Concentrator) generates a strong magnetic field along z direction to compensate the transverse momentum of the positron. 36 1.3 m L-band standing wave accelerators with 0.5 Tesla solenoid field are placed for positron capture. This section is called as Positron Capture Linac. At the downstream of Positron Capture Linac, a chicane is placed to remove electrons. The positron booster is composed from 2.0 m L- and 2.0 m S-band traveling wave accelerators. ECS is composed from 2.0 m L-band traveling wave accelerators with chicanes.

In E-Driven ILC positron source, positrons are generated in a multi-bunch format as shown in Figure 2. It contains 66 bunches with 80 ns gap. To generate 1312 bunches for positrons in one RF pulse in the main linac, the positron

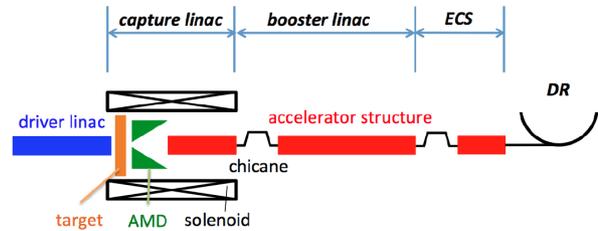


Figure 1: Configuration of E-Driven ILC positron source is schematically shown.

generation is repeated 20 times in 64 ms. The positron is stored in DR for 135 ms before the acceleration at the main linac for collision. Because the positron is generated over 64 ms, the instantaneous heat load on the target is much suppressed [3]. The pulse format shown in Figure 2 is identical to a part of the DR fill pattern [4].

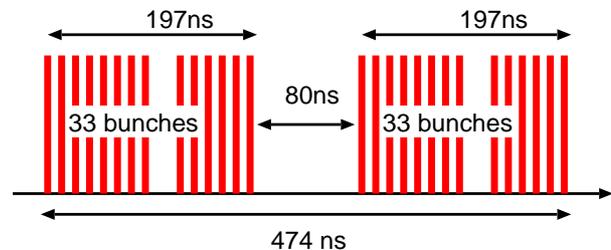


Figure 2: The beam structure in the positron source. Each mini-train contains 33 bunches. Each pulses contain 2 or 1 mini-trains.

A first simulation for the injector part is made by T. Omori [3]. A simulation with the tracking down to DR was made by Y. Seimiya [5], but no beam loading effect was accounted. A new simulation with the beam-loading effect was done by Kuriki and Nagoshi [6] [7]. For those simulation, the peak energy deposition density on the target is kept less than 35 J/g [8], which is considered to be a practical limit of the target destruction.

The generated positron has a large spread in both longitudinal and transverse momentum space. Capturing the positron in an RF bucket for further acceleration is the role of the capture linac. Deceleration capture was proposed by M. James et al. [9] for better capture efficiency. In this method, the positrons are placed on a deceleration phase and move to the acceleration phase by phase-slipping. As the result, the positron phase space distribution is large in

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longitudinal momentum, but small in longitudinal space ( $z$ ). By boosting the positrons further, the longitudinal momentum spread is suppressed resulting a good capture efficiency.

This deceleration capture cause a difficulty on the beam loading compensation, because the beam phase (RF phase where the beams is) is moving from  $-\pi$  to 0. It causes a problem on the beam loading compensation, because the conventional theory assumes the beam phase is constant.

## BEAM LOADING CCMPENSATION WITH A STANDING WAVE LINAC

The acceleration voltage by a standing wave RF accelerator with the beam loading is

$$V(t) = \frac{2\sqrt{\beta PrL}}{1+\beta} \left(1 - e^{-\frac{t}{\tau}}\right) - \frac{IrL}{1+\beta} \left(1 - e^{-\frac{t-t_b}{\tau}}\right) e^{i\theta} \quad (1)$$

where  $\beta$  is coupling beta,  $P$  is input RF power,  $r$  is shunt impedance,  $L$  is structure length,  $\tau = 2Q/\omega/(1+\beta)$ ,  $I$  is beam loading current,  $t_b$  is timing to start the beam acceleration, and  $\theta$  is relative phase of the beam center to the RF. Here, we omit the RF oscillation term,  $e^{i\omega t}$ .

Both terms, RF input and beam field, is growing as  $1 - e^{-t/\tau}$ . If  $\theta = 0$  and  $t_b$  is adjusted properly as

$$t_b = -\ln\left(\frac{I}{2}\sqrt{\frac{rL}{\beta P}}\right), \quad (2)$$

the voltage variations by the RF term and the Beam term have the same amplitude and opposite sign, and  $V(t)$  can be a constant.

If  $\theta$  is a finite value, there is no solution with this method. Figure 3 shows these components as a phase diagram. The RF term shown as a blue arrow is set as 0 phase and the beam term shown as a read arrow has a phase  $\theta$ . The solid arrow is the instantinuous field and the blurred arrow is the asymptotic value. The cavity voltage can't be a constant.

To compensate the voltage variation by the beam term, the RF term should contain a component which has a same aplitude and anti-sign as the beam term. In addition, the RF term should have a component to maintain a constant cavity voltage. The constant cavity voltage should be same as  $V(t_b)$ , which is the RF term value when we start the beam acceleration. This condition is satisfied if the asymptotic value is set to  $V(t_b)$ . The condtion is

$$V_{RF} e^{i\phi} + V_B e^{i\theta} = V_{RF0} \quad (3)$$

where  $V_{RF}$  is the asymptotic value of the RF term and we assume

$$V_{RF} = \frac{2\sqrt{\beta PrL}}{1+\beta}, \quad (4)$$

i.e. we keep the same input RF power, but a phase modulation with  $\phi$ .  $V_B$  is the asymptotic value of the beam term

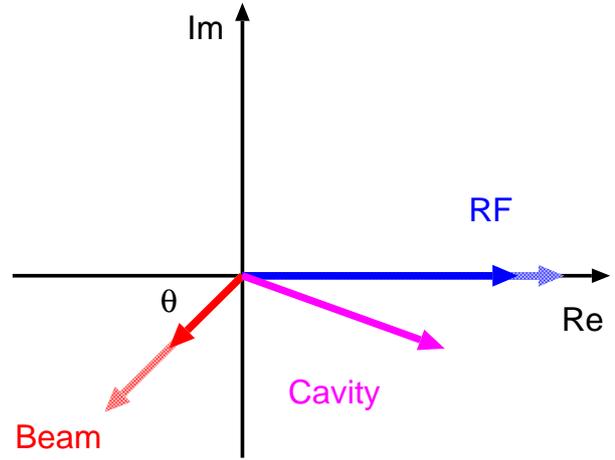


Figure 3: Phase diagram of RF term (blue) and Beam term (red) with phase  $\theta$ . The solid and hatched arrows show the instantinuous and the asymptotic values.

as

$$V_B = -\frac{rLI}{1+\beta}. \quad (5)$$

From this condion, the phase modulation on the input RF  $\phi$  is determined as

$$\phi = \sin^{-1}\left(-\frac{V_B}{V_{RF}} \sin \theta\right) \quad (6)$$

and  $V_{RF0}$  is

$$V_{RF0} = \sqrt{V_{RF}^2 + V_B^2 \sin^2 \theta} + V_B \cos \theta \quad (7)$$

$t_b$  is obtained as

$$t_b = -\tau \ln\left(1 - \frac{V_{RF0}}{V_{RF}}\right) \quad (8)$$

Figure 4 shows the phase diagram with the phase mulation  $\phi$  on the input RF. The circle corresponds to the amplitude of the input RF. The phase modulation is determined as  $V_{RF} e^{i\phi} + V_B e^{i\theta}$  giving  $V_{RF0}$ . The RF term is growing towards the asymptotic value shown as the blue hatched arrow in Figure 4. The beam term is also growing towards the asymptotic value shown as the red hatched arrow in Figure 4. Because these components have the sampe amplitude and opposite sign varying with the same time constant, the variation is completely cancelled.

If  $\theta > \pi/2$ , the solution is different, because  $V_B$  gives acceleration. In this case,  $V_{RF0}$  is

$$V_{RF0} = V_{RF} e^{i\phi} + V_B e^{i\theta} \quad (9)$$

and is larger than  $V_{RF}$ , but it is impossible. Instead, we change the RF input power as  $V_{RF1}$  after the acceleration. In this case

$$V_{RF0} = V_{RF1} e^{i\phi} + V_B e^{i\theta} \leq V_{RF} \quad (10)$$

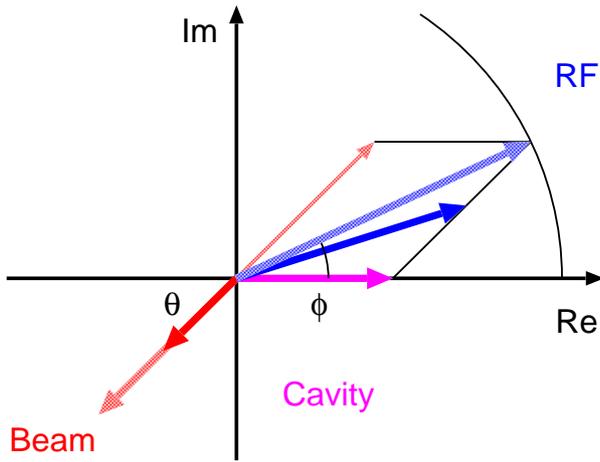


Figure 4: Phase diagram of RF term (blue) and Beam term (red) with phase  $\theta$  after the phase modulation of the input RF with  $\phi$ . The solid and hatched arrows show the instantaneous and the asymptotic values.

if we take the equal as the maximum acceleration,

$$V_{RF0} = V_{RF}, \quad (11)$$

and

$$V_{RF1}^2 = V_{RF}^2 + V_B^2 - 2V_B V_{RF} \cos \theta. \quad (12)$$

The phase modulation  $\phi$  is

$$\phi = \sin^{-1} \left( -\frac{V_B}{V_{RF1}} \sin \theta \right). \quad (13)$$

Figure 5 shows the phase diagram with  $\theta > \pi/2$ . To obtain  $V_{RF0}$  which is equal to the asymptotic value, we have to wait forever, but it is not practical. The input power after the acceleration should be less than  $V_{RF}$  as shown in Figure 5

## BEAM LOADING COMPENSATION AT THE PULSE GAP

ILC pulse has a gap in a pulse and the duration is 80 ns as shown in Figure 2. In the gap, the balance between the RF term and the beam term isn't maintained because the beam term amplitude becomes zero. We want keep  $V_{RF0}$  without  $V_B$  in the gap. The solution is very simple, the input RF power should be modulated as giving  $V_{RF0}$  as the asymptotic value. The input power  $P_1$  should be

$$P_1 = \frac{(1 + \beta)^2 (\sqrt{V_{RF}^2 - V_B^2 \cos^2 \theta} + V_B \sin^2 \theta)^2}{4\beta r L} \quad (14)$$

At the gap end, the input RF should be back to Eq. (6) and (7). The beam voltage and the counter part by the input RF are growing again, but they are cancelled to each other. This cancellation is maintained if the input RF is switched in a same manner.

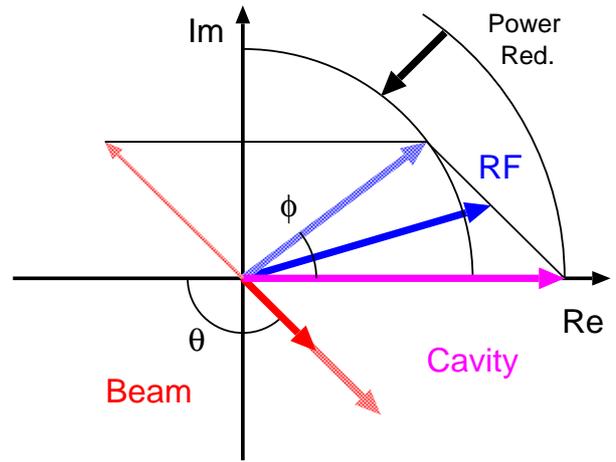


Figure 5: Phase diagram of RF term (blue) and Beam term (red) with phase  $\theta$  after the phase modulation of the input RF with  $\phi$ . The solid and hatched arrows show the instantaneous and the asymptotic values. If  $\theta > \pi/2$ , the input power should be also modulated (less power).

To demonstrated the beam loading compensation with the pulse gap, we calculated the cavity voltage at the various condition. Cavity voltage is calculated with a simple sigle cell model, where the cavity as a treated one RF cell. The differential equation for the cavity voltage  $V$  is given as

$$\frac{dV}{dt} = -\frac{(1 + \beta)\omega}{2Q} V - \frac{\omega r L I}{2Q} + \frac{\omega}{Q} \sqrt{\beta P r L}, \quad (15)$$

where  $\omega$  is angular frequency of RF, and  $Q$  is Q value. The first term shows the power dissipation, the second term shows the beam loading, and the last term shows the input RF.

As the beam current, we assume pulses with intervals as shown in Figure 6. The beam loading current is 1A.

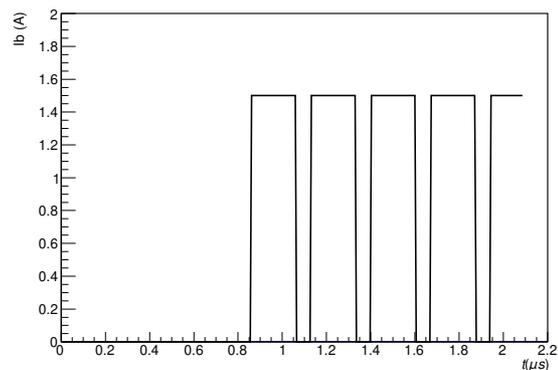


Figure 6: The pulse structure assumed for the calculation. The pulse contains 200 ns mini-pulses and 80 ns gaps.

Figure 7 shows the cavity voltage variation with the phase modulation determined by Eq. (6) and (7), but no treatment on the gap. The beam phase  $\theta = \pi/6$  in this calculation. The

solid and dashed lines show the real and imaginary parts. The cavity voltage is kept as a constant when the beam acceleration is started, but the cavity voltage is increased in the gap. The imaginary part is increased to the negative corresponds to the beam phase. The cavity voltage is decreased if the beam is back.

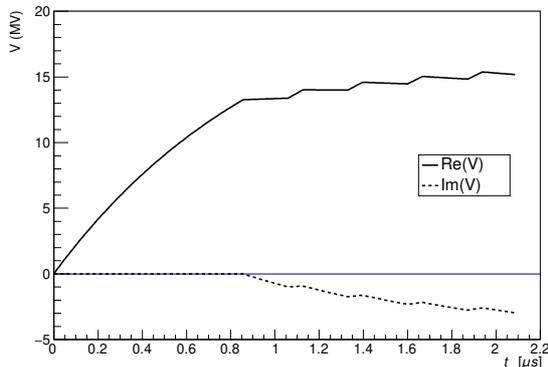


Figure 7: Cavity voltage variation with the input RF phase modulation, but no treatment on the gap. The cavity voltage is increased in the gap.

Figure 9 shows the cavity voltage variation with the phase modulation by Eq. (6) and (7), and the gap treatment by Eq. (7). The solid and dashed lines show the real part and imaginary part. There is no imaginary part amplitude at all. This shows the beam term is perfectly cancelled by the input RF modulation.

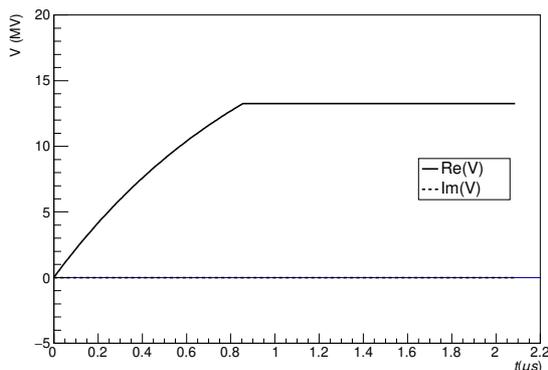


Figure 8: Cavity voltage variation with the input RF phase modulation including the amplitude modulation on the gap. The cavity voltage is maintained as a constant. The solid and dashed lines are the real and imaginary components. The imaginary components is not appeared at all.

Figure 9 and 10 show the real part and imaginary part of RF term (red lines) and beam term (blue lines). The imaginary parts of RF term and beam term are symmetric on the axis.

The same demonstration was done by S. Konno [10] [11] with a multi-cell coupled resonator model developed by T.

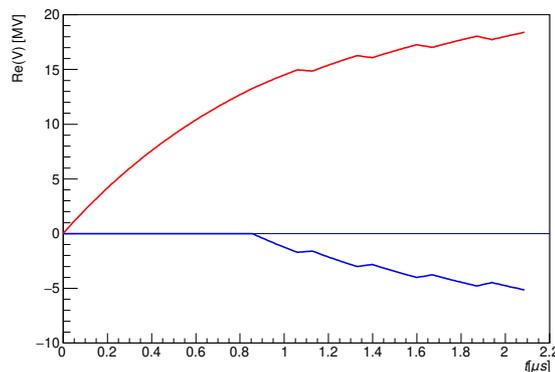


Figure 9: The real part of RF term (red) and Beam term (blue) with the input RF phase modulation and the amplitude modulation on the gap. RF term and Beam term are changing in time depending on the pulse structure, but their amplitude are same with different sign.

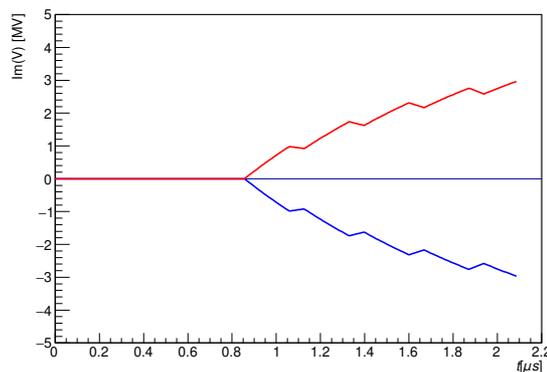


Figure 10: The imaginary part of RF term (red) and Beam term (blue) with the input RF phase modulation and the amplitude modulation on the gap. The RF term and beam term repeated the cycle of growth and decay depending on the pulse structure. Their amplitude are same with different sign.

Shintake [12]. The compensation was as excellent as that by the single cell model. Because the multi-cell model is more realistic than the single cell model, the result shows the reliability of this compensation method.

## INPUT RF MODULATION

The cavity voltage variation including the pulse gap is perfectly compensated by the input RF phase and amplitude modulation as demonstrated in the previous section. Here, we consider how the modulation is implemented.

A direct method to implement AM and FM on the input RF is AM and FM on the input RF signal to klystron. This method is not ideal because the response (amplification) is not linear and changing the input RF signal amplitude causes a beat-wave.

Instead of the direct modulation, we consider a combination of two input RF signals with a constant RF amplitude. The phase of each input RF signals are modulated. If the phase modulation for the two klystrons are same sign, the combined RF is

$$V_{RF}e^{i\phi} + V_{RF}e^{i\phi} = 2V_{RF}e^{i\phi}. \quad (16)$$

This is a phase modulation with  $\phi$ . If the sign are opposite,

$$V_{RF}e^{i\phi} + V_{RF}e^{-i\phi} = 2V_{RF}\cos\phi. \quad (17)$$

This is an amplitude modulation with  $\cos\phi$ . If we want the phase and amplitude modulation simultaneously, the modulation should be

$$V_{RF}e^{i(\phi_1+\phi_2)} + V_{RF}e^{i(\phi_1-\phi_2)} = 2V_{RF}e^{i\phi_1}\cos\phi_2. \quad (18)$$

resulting phase modulation with  $\phi_1$  and amplitude modulation with  $\cos\phi_2$ .

## CONCLUSION

We consider the beam loading compensation for the positron capture linac for ILC E-Driven positron source. Due to the heavy beam loading, its compensation is essential to obtain an uniform intensity positron pulse. We consider PM and AM on the input RF to compensate the beam loading with a finite phase to the input RF. It works well including the gap in a pulse. AM and PM can be implemented by combining two RF with PM on the input signal.

## ACKNOWLEDGEMENTS

This work is partly supported by Grant-in-Aid for Scientific Research (B) and US-Japan Science and Technology Cooperation Program in High Energy Physics.

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